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Differential Geometry II: Analysis and Geometry on Manifolds

Exercise Sheet 2

(product manifolds, smooth maps, embeddings)

due 01.11.2011

Exercise 1 **5 points**

Let M and N be smooth manifolds of dimension m and n respectively. The topological product $M \times N$ is a topological $(m+n)$ manifold. Let $(U_i, \varphi_i)_{i \in I}$ be a smooth atlas for M and $(V_j, \psi_j)_{j \in J}$ a smooth atlas for N.

Show that $(U_i \times V_j, \varphi_i \times \psi_j)_{(i,j) \in I \times J}$ is a smooth atlas for $M \times N$.

Here $\varphi_i \times \psi_j \colon U_i \times V_j \to \mathbb{R}^m \times \mathbb{R}^n$ with $(\varphi_i \times \psi_j)(x, y) = (\varphi_i(x), \psi_j(y))$.

Exercise 2 **5 points**

Let M_1, M_2 and N be smooth manifolds and $f \colon N \to M_1 \times M_2$ be a map. For $i \in \{1, 2\}$ we define on the projection on the $i-th$ component:

$$\pi_i \colon M_1 \times M_2 \to M_i, \qquad \pi_i(x_1, x_2) := x_i.$$

Prove that f is smooth if and only if $\pi_1 \circ f$ and $\pi_2 \circ f$ are smooth.

Exercise 3 **5 points**

Let X, Y be Hausdorff spaces and $f \colon X \to Y$ a continuous map. Prove the equivalence of the following statements:

i) For each $x \in X$ and every neighborhood U of x there is a neighborhood V of $f(x)$ such that $f(X \setminus U) \cap V = \emptyset$.

ii) f is a topological embedding, i.e. a continuous injection which is a homeomorphism onto its image.