TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik
Sullivan / Knöppel
http://page.math.tu-berlin.de/~sullivan/L/11W/DG2/ WS 11

Differential Geometry II: Analysis and Geometry on Manifolds

Exercise Sheet 13

(Ricci curvature, Lie groups)

due 8.2.2012

Exercise 1 **5 points**

Let ω be the canonical volume form on $\mathbb{S}^{n-1}$. Prove that the scalar curvature $S(p)$ at $p \in M^n$ is given by

$$S(p) = \frac{1}{\operatorname{vol}(\mathbb{S}^{n-1})} \int_{\mathbb{S}^{n-1}} \operatorname{Ric}_p(x, x)\omega.$$

Exercise 2 **5 points**

Let $SO(n) := \{A \in Gl(n) \mid AA^T = I, \det A = 1\}$. Show that

i) $SO(n)$ is a Lie group,

ii) $\mathfrak{so}(n) := T_I SO(n) = \{X \in \mathfrak{gl}(n) \mid X^T = -X\}$ and $T_A SO(n) = A \cdot \mathfrak{so}(n) = \mathfrak{so}(n) \cdot A$,

iii) $g(X, Y) := \operatorname{tr}(XY^T)$ defines a Riemannian metric on $SO(n)$,

iv) the left multiplication L_A and the right multiplication R_A are isometries of $SO(n)$.

What is the dimension of $SO(n)$?

Exercise 3 **5 points**

Show that, for any Lie group G, the left multiplications L_a satisfy

$$D_b L_a = D_e L_{ab} \circ (D_e L_b)^{-1},$$

where $D_b L_a \colon T_b G \to T_{ab} G$. State and prove a similar formula for right multiplications R_a.