Exercise Sheet 9
Due in tutorials on 5 January 2010

Exercise 1 (5 pts):

Show that the fundamental group of the closed disk D^2 is trivial.

Fill in the details in the proof that winding number provides an isomorphism from $\pi_1(\mathbb{S}^1, *)$ to $\mathbb{Z}$.

Use this to give a new proof that there is no retraction from D^2 to $\mathbb{S}^1$.

Exercise 2 (5 pts):

Show that
$$\pi_1\big(X \times Y, \{x_0, y_0\}\big) \cong \pi_1(X, x_0) \times \pi_1(Y, y_0).$$

Exercise 3 (5 pts):

Show that being homotopy equivalent is indeed an equivalence relation on spaces.

A retraction from a space X onto a subspace Y is called a *deformation retraction* if it is homotopic to the identity on X.

Show then that X and Y are homotopy equivalent and conclude that the retraction induces an isomorphism of fundamental groups

Exercise 4 (5 pts):

If $f : \mathbb{S}^n \to \mathbb{S}^n$ has no fixed points, show that f is homotopic to the antipodal map $p \mapsto -p$.