Exercise Sheet 7
Due in tutorials on 8 December 2010

Exercise 1 (6 pts):

Suppose $p : Y \to X$ and $p' : Y' \to X$ are connected coverings, with X furthermore locally connected.
Suppose there is a continuous map $q : Y \to Y'$ such that $p' \circ q = p$.
Show that q is a covering map.

Exercise 2 (7 pts):

Suppose X is the "wedge of two circle" and Y is the space consisting of four circles joined as in figure.
Let $p : Y \to X$ be the mapping taking each of the three arcs labeled α to the circle α, and similarly for β.
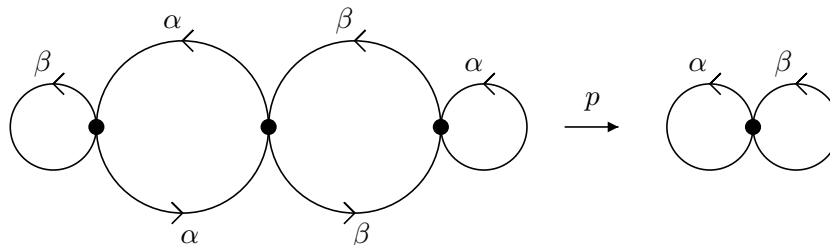Show that p is a three-sheeted covering.
Let γ be the closed path in X given by $\alpha * \beta * \overline{\alpha} * \overline{\beta}$, that is, follow α, then β, then α in reverse, then β in reverse.
What are the three liftings of γ to Y?
Deduce that γ is not homotopic in X to a constant path.
Show however that γ is homologous to zero.

Exercise 3 (7 pts):

Let $I^n = [0, 1]^n$ be an n-dimensional rectangle.
Prove, using the Homotopy Lifting theorem and induction on n, that any covering on I^n is trivial.