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Topology
WS 10/11

Exercise Sheet 4

Due in tutorials on 17 November 2010

Exercise 1:

Suppose $A \subset \mathbb{R}^2$ is connected and closed, and $P \in A$.
Show that $[\omega_P] = 0 \in H^1(\mathbb{R}^2 \setminus A)$ if and only if A is unbounded.

Exercise 2:

Suppose U and V are connected, open subsets of $\mathbb{R}^2$.
Show that if $H^1(U \cup V) = 0$ then $U \cap V$ is also connected.

Exercise 3:

Suppose $X \subset \mathbb{R}^2$ is homeomorphic to a figure-8, that is to two circles sharing a point.
Show that $\mathbb{R}^2 \setminus X$ has exactly three components.

Exercise 4:

Let $P_1, \dots P_n$ be distinct points of the plane and set $U = \mathbb{R}^2 \setminus \{P_1, \dots P_n\}$.
Describe $H^0(U)$ and $H^1(U)$.