TECHNISCHE UNIVERSITÄT BERLIN Institut für Mathematik

Berlin Mathematical School

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Exercise Sheet 3

Due in tutorials on 10 November 2010

Exercise 1:

We say that a space X has the *fixed-point property* if every map $f : X \to X$ has a fixed point. Show that if X has the fixed-point property and $f : X \to Y$ is a retraction onto a subspace $Y \subset X$, then also Y has the fixed point property.

Exercise 2:

If $f: \mathbb{S}^1 \to \mathbb{R}^2 \smallsetminus \{0\}$ has W(f, 0) = 0, show that f can be extended to a continuous map $\overline{f}: D^2 \to \mathbb{R}^2 \smallsetminus \{0\}.$

(Hint: Each path with winding number zero, such as f, is homotopic to a constant path.)

Exercise 3:

Suppose $f : \mathbb{S}^1 \to \mathbb{S}^1$ has no fixed point. Show that $\deg(f) = 1$, and thus that f is surjective.

Exercise 4:

Suppose $f : \mathbb{S}^1 \to \mathbb{S}^1$ is even in the sense that f(-p) = f(p). Show that $\deg(f)$ is even.

Exercise 5:

Suppose $f : \mathbb{S}^1 \to \mathbb{S}^1$ never has the same value at two antipodal points (that is, $f(-p) \neq f(p)$ for all p). Show that f is surjective.