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Topology
WS 10/11

Exercise Sheet 2

Due in tutorials on 3 November 2010

Exercise 1:

Let $U \subset \mathbb{R}^2$ be an open subset in the plane. Prove that a function $f : U \to \mathbb{R}$ is *locally constant* (i.e. each point has a neighborhood on which f is constant) if and only if f is constant on each connected component.

Exercise 2:

On which of the following open subsets $U \subset \mathbb{R}^2$ in the plane is ω_θ exact (i.e. there is a smooth angle function θ with $\omega_\theta = d\theta$ on U)? Motivate your answer.

(a) The union of the upper half plane and the right half plane.

(b) The complement of the negative y-axis.

(c) The complement of the line segment joining P to 0, where $0 \neq P \in \mathbb{R}^2$.

(d) The complement of the line through P and 0, where again $0 \neq P \in \mathbb{R}^2$.

(e) An annulus centered at 0, that is, the set $\{(x, y) \in \mathbb{R}^2 \mid n < x^2 + y^2 < m\}$, for some $0 < n < m$.

Exercise 3:

Let $P \in \mathbb{R}^2$, then ω_P will be the closed 1-form corresponding to "$d\theta$"around P. If $P = (x_0, y_0)$ then we'll define it as follow on $\mathbb{R}^2 \setminus \{P\}$:

$$\omega_P := \frac{-(y - y_0)dx + (x - x_0)dy}{(x - x_0)^2 + (y - y_0)^2}$$

Given two distinct points $P,\ Q \in \mathbb{R}^2$ prove that $\omega_P - \omega_Q$ is not exact on $\mathbb{R}^2 \setminus \{P, Q\}$. Show, however, that it is exact on $\mathbb{R}^2 \setminus \overline{PQ}$, where $\overline{PQ}$ denotes the line segment from P to Q.

Exercise 4:

Suppose $U \subset \mathbb{R}^2 \setminus \{0\}$ is a subset in which there exists a continuous angle function θ (for which $d\theta = \omega_\theta$). Suppose γ is a closed path in U. Show that the winding number $W(\gamma, 0)$ equals 0.