Exercise Sheet 13
Due in tutorials on 2 February 2011

In these days the angel of topology and the devil of abstract algebra fight for the soul of each individual mathematical domain.

Hermann Weyl, 1939

Exercise 1 (5 pts):

If $p : Y \to X$ is a covering, then the induced map on fundamental groups is injective. How about the induced map on H_1? Prove or give a counter-example.

Exercise 2 (5 pts):

Let X be the discrete two-point space. Calculate the homology groups $H_*(X)$.

Exercise 3 (5 pts):

Check the functoriality of the homology functor H_*.
That is, if $f \colon X \to Y$ and $g \colon Y \to Z$ are maps of topological spaces, then the induced maps on homology satisfy $(f \circ g)_* = f_* \circ g_*$.

Exercise 4 (5 pts):

Prove from the Eilenberg–Steenrod axioms that for any space X we have $H_p(X, X) = 0$ for all p.