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Topology
WS 10/11

Exercise Sheet 12

Due in tutorials on 26 January 2011

Exercise 1 (5 pts): Let $P = \mathbb{S}^2/\pm$ be the projective plane. Let $U = P \setminus \{p\}$ be the complement of a single point in P. (U is homeomorphic to a Möbius band.) What is $\pi_1(U)$?

Exercise 2 (15 pts):

Suppose X is the figure-eight space, consisting of circles A and B joined at the basepoint x_0. Its fundamental group is

$$\pi_1(X, x_0) = \langle A \rangle \star \langle B \rangle \cong \mathbb{Z} \star \mathbb{Z}.$$

1. Consider the infinite cyclic subgroup $H < \pi_1(X)$ generated by the element A^2. What is the covering space of X corresponding to this subgroup H? (Don't try to give a rigorous proof. Just sketch the space and explain why your sketch is correct.) What is the automorphism group of this cover?

2. Now consider the map from $\pi_1(X)$ to the abelian group $\mathbb{Z}/3 \oplus \mathbb{Z}/3$ which takes A to $(1, 0)$ and B to $(0, 1)$. The kernel of this map is a normal subgroup K of π_1. What is the cover of X corresponding to this subgroup K? (Again, sketch the cover and explain why it corresponds to this kernel, without giving a full proof.) What is the automorphism group of this cover?