# 13. Übung Differentialgeometrie II: Mannigfaltigkeiten

(curvature, Poincare half plane part 3 )

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## Hausaufgaben

### 1. Aufgabe (5 Punkte)

Let $k \in \mathbb{R}$ and $M_k := \{p \in \mathbb{R}^2 : 1 + k|p|^2 > 0\}$ and $g_k(p) := \frac{4}{(1+k|p|^2)^2} g_0$, where $g_0$ is the canonical metric on $\mathbb{R}^2$, a Riemannian metric on $M_k$. Show that the sectional curvature on $(M_k, g_k)$ is constant and equal to $k$. Are these manifolds isometric to each other?

### 2. Aufgabe (5 Punkte)

Show that the sectional curvature of the Poincare half plane $\mathbb{H}^2$ is equal to $-1$.

### 3. Aufgabe (5 Punkte)

Define a connection on $M := \mathbb{R}^3$ by setting

$$\Gamma^3_{12} = \Gamma^1_{23} = \Gamma^2_{31} = 1$$
$$\Gamma^3_{21} = \Gamma^1_{32} = \Gamma^2_{13} = -1,$$

and all other Christoffel symbols to zero. Show that this connection is compatible with the Euclidean metric, but it is not symmetric. Compute the connection 1-forms $\omega^j_i$, curvature 2-forms $\Omega^j_i$ and all sectional curvatures with respect to the canonical orthonormal basis.

Gesamtpunktzahl: 15