TECHNISCHE UNIVERSITÄT BERLIN WS 10
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http://www.math.tu-berlin.de/~sullivan/L/10W/DG2

12. Übung Differentialgeometrie II: Mannigfaltigkeiten

(connections, parallel transport, Pioncare half plane model part 2)

Hausaufgaben

1. Aufgabe (5 Punkte)

Let (M, g) and (N, h) be a Riemannian manifold equipped with the Riemannian connections ∇ and $\tilde{\nabla}$, respectively. Let $f : M \to N$ be an isometry. Show:

1. f maps the Riemannian connection ∇ to the Riemannian connection $\tilde{\nabla}$ in the sense

$$\nabla_X Y = (df^{-1})(\tilde{\nabla}_{df(X)} df(Y));$$

2. f maps the covariant derivative $\frac{D}{dt}$ on M to the covariant derivative $\frac{\tilde{D}}{dt}$ on N, in the sense that if $c : I \to M$ is a curve in M, X is a vector field along c, $\tilde{c} := f \circ c$ and $(df)(X)$ is the corresponding vector field along $\tilde{c}$, then

$$\frac{DX}{dt} = (df^{-1})\frac{\tilde{D}df(X)}{dt};$$

3. f maps geodesics in M into geodesics in N: if γ is a geodesic in M, then $f \circ \gamma$ is a geodesic in N.

2. Aufgabe (5 Punkte)

Let $c : I \to M$ be a curve in M and $c(t_0)$ be a point on the curve. The mapping $P_{c(t),c(t_0)} : T_{c(t_0)}M \to T_{c(t)}M$ defined by $P_{c(t),c(t_0)}V_{c(t_0)} = V_{c(t)}$, and $V_{c(t)}$ is the unique extension of $V_{c(t_0)}$ to a parallel vector field along c, is called the parallel transport from $c(t_0)$ to $c(t)$. Show:

1. The parallel transport is a linear isomorphism. If $V(c(t))$ is a vector field along c, then

$$\lim_{t \to 0} \frac{P_{c(t_0),c(t)}V_{c(t)} - V_{c(t_0)}}{t - t_0} = \frac{DV}{dt}.$$

2. if M is a Riemannian oriented manifold with the Riemannian connection ∇ then $P_{c(t),c(t_0)}$ is an isometry which preserves the orientation.

3. Aufgabe

(5 Punkte)

Show that the geodesics of the Pioncare half plane $\mathbb{H}^2$ are vertical lines and semi-circles with the center on the x-axis.

Gesamtpunktzahl: 15