TECHNISCHE UNIVERSITÄT BERLIN WS 10
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http://www.math.tu-berlin.de/~sullivan/L/10W/DG2

1. Übung Differentialgeometrie II: Mannigfaltigkeiten

(Definition of manifold, examples)

Hausaufgaben

1. Aufgabe (5 Punkte)

Let X be a topological space, $x \in X$ and $n \geq 0$. Show that the following statements are equivalent:

1. There is a neighbourhood of $x \in X$ which is homeomorphic to $\mathbb{R}^n$.
2. There is a neighbourhood of $x \in X$ which is homeomorphic to an open subset of $\mathbb{R}^n$.

2. Aufgabe (5 Punkte)

Let $n \geq 0$ and J be the set $\{0, \ldots, n\} \times \{-1, 1\}$. We define $(U_j, \phi_j)_{j \in J}$ by

$$U_{k,i} := \{x \in \mathbb{S}^n : ix_k > 0\}$$

and

$$\begin{aligned} \phi_{k,i} : U_{k,i} &\to \mathbb{R}^n \\ x = (x_0, \ldots, x_n) &\mapsto (x_0, \ldots, x_{k-1}, x_{k+1}, \ldots, x_n) \end{aligned}$$

Show that $(U_j, \phi_j)_{j \in J}$ is a C^∞-atlas for $\mathbb{S}^n$.

3. Aufgabe (5 Punkte)

For $\phi \in \mathbb{R}$ we define $U_\phi := \mathbb{S}^1 \backslash \{(\cos(\phi), \sin(\phi))\}$

$$\begin{aligned} f_\phi \colon U_\phi := &\to (\phi, \phi + 2\pi) \\ (\cos(\alpha), \sin(\alpha)) &\mapsto \alpha \end{aligned}$$

1. Explain why this is well-defined and show that $(U_\phi, f_\phi)_{\phi\in\mathbb{R}}$ is a C^∞-atlas for $\mathbb{S}^1$.

2. Show that

$$\begin{aligned} f : \{(x_0, x_1) \in \mathbb{S}^1 : x_1 > 0\} &\to \mathbb{R} \\ (x_0, x_1) &\mapsto \frac{x_0}{x_1} \end{aligned}$$

   is another coordinate chart for $\mathbb{S}^1$ and decide whether it is in the C^∞-structure defined by the atlas $(U_\phi, f_\phi)_{\phi\in\mathbb{R}}$. Are these structures diffeomorphic?

Gesamtpunktzahl: 15