TECHNISCHE UNIVERSITÄT BERLIN
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Geometry II
SS 10

http://www.math.tu-berlin.de/~sullivan/L/10S/Geo2/

Exercise Sheet 6

Exercise 1: Reuleaux triangles. (4 pts)

A *Reuleaux triangle* is the planar body of constant width made from three circular arcs centered at the corners of an equilateral triangle. Glue four Reuleaux triangles along their edges in tetrahedral fashion. (This can be done in 3-space, say with paper models, or we can think about the resulting piecewise-smooth surface intrinsically.) What is the Guass curvature?

Exercise 2: CMC surfaces. (4 pts)

If p is a vertex on a polyhedral surface M in $\mathbb{R}^3$, we defined the area vector A_p and the mean curvature vector H_p as the gradients of volume and surface area. We said that M is a *discrete CMC surface* if there is a constant λ such that $H_p = \lambda A_p$ for all p.

Consider a triangular bipyramid M with three vertices equally spaced around the unit xy-circle and two more at heights $\pm h$ along the z-axis. For what value(s) of h is M a discrete CMC surface?

Exercise 3: Convex polytopes. (4 pts)

Let $P \subset \mathbb{R}^3$ be a convex polytope containing the origin O. For a facet F of P, denote by $\alpha(F)$ the sum of the angles of F and by $\beta(F)$ the sum of the angles of the projection of F onto a unit sphere centered at O. Finally, let $\omega(F) = \beta(F) - \alpha(F)$. Prove that

$$\sum_{F \subset P} \omega(F) = 4\pi.$$

Turn over

Exercise 4: Equihedral tetrahedra. (6 pts)

Let $\triangle \subset \mathbb{R}^3$ be a tetrahedron. Prove that the following conditions are equivalent:

1. All faces of $\triangle$ are congruent triangles;
2. All faces of $\triangle$ have equal perimeter;
3. All vertices of $\triangle$ have equal curvature;

Such tetrahedra are called *equihedral tetrahedra*.

Prove that a tetrahedron has three pairwise intersecting simple closed geodesics if an only if it is equihedral.

Due: Tutorial on 15.07.10