TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

Berlin Mathematical School

Prof. Dr. John M. Sullivan
Dott. Matteo Petrera

Geometry II
SS 10

http://www.math.tu-berlin.de/~sullivan/L/10S/Geo2/

Exercise Sheet 4

Exercise 1: Distortion of an infinite polygon. (6 pts)

Given $\epsilon > 0$ find an "infinite polygon" $p = (p_1, p_2, ...)$ in $\mathbb{R}^2$ with $p_n \to (0,0)$ such that $TC(p) = \infty$ and $\delta(p) < 1 + \epsilon$.

(Hint: Let the vertices alternate between two nearby rays out of $(0,0)$).

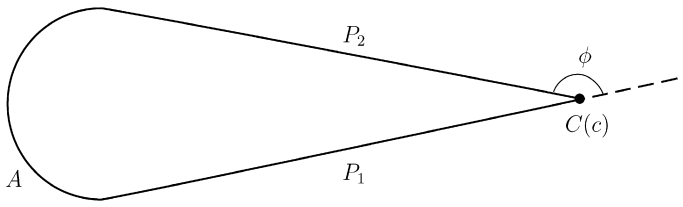
Exercise 2: Schur's Theorem. (6 pts)

Let $\gamma \subset \mathbb{R}^d$ be an arc with length $2a$, $a < \pi$. Suppose γ has curvature $\kappa \leq 1$. Use Schur's Theorem to show that the distortion of γ is at most $a/\sin a$.

(Note: The condition $\kappa \leq 1$ should be interpreted as saying that the total curvature of any subarc is at most its length. For a C^2 curve, this is equivalent to saying that the ordinary curvature $\kappa := |T'|$ is bounded by 1.)

Exercise 3: Distortion of a comet shaped curve. (4 pts)

Let $\gamma : S^1 \to \mathbb{R}^2$ be the closed curve in the figure, consisting of a semicircle A and a polygonal section, consisting of two line segments P_1 and P_2 with the same length. Let ϕ be the exterior angle. Compute the distortion $\delta(\gamma)$.



Due: Tutorial on 27.05.10