TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

Berlin Mathematical School

Prof. Dr. John M. Sullivan
Dott. Matteo Petrera

Geometry II
SS 09/10

http://www.math.tu-berlin.de/~sullivan/L/10S/Geo2/

Exercise Sheet 1

Exercise 1: Functions of bounded variation. (6 pts)

Let $f : [0, 1] \to \mathbb{R}$ be a bounded real function. Give examples of functions f which are:

1. continuous but not of bounded variation (BV);
2. BV but not continuous;
3. regulated but neither BV nor continuous.

Exercise 2: The Schwarz lantern. (6 pts)

Consider the unit cylinder $C := \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = 1,\ 0 \le z \le 1\} \subset \mathbb{R}^3$ of surface area 2π. The *Schwarz lantern* is an inscribed polyhedron, say $C_{m,n}$, depending on parameters m, n. We take $m(n+1)$ vertices, a regular m-gon at each height k/n, but staggered so that the vertices at even levels are at angles $2\pi j/m$ while those at odd levels are at angles $\pi(2j + 1)/m$. The polyhedron $C_{m,n}$ is built of $2mn$ congruent isosceles triangles.

1. Find the area of $C_{m,n}$ as a function of m, n;
2. Show that any limiting area greater than or equal to 2π can be achieved in some limit of $m, n \to \infty$.
3. Show that if the shapes of the triangles are bounded (say, if the angles are never smaller than some $\epsilon > 0$) as $m, n \to \infty$, then the area converges to 2π.

Due: Tutorial on 29.04.10