Differentialgeometrie II
Übungsblatt 4

Due 26 November 2008

1 Aufgabe

Let $M = \mathbb{R}^2$ and $\theta : \mathbb{R} \times M \to M$ be given by the formula

$$\theta_t(x, y) = (x \cos t + y \sin t, -x \sin t + y \cos t).$$

- Show that θ is a globally defined action of $\mathbb{R}$ on M.
- Describe X, the associated infinitesimal generator.
- Describe the orbits.
- Show explicitly that X is invariant with respect to θ, i.e., that $\theta_t^*(X_{(x,y)}) = X_{\theta_t(x,y)}$.

2 Aufgabe

Let $M = \mathbb{R}^2$, the x, y plane, and $X = y(\frac{\partial}{\partial x}) + x(\frac{\partial}{\partial y})$. Find the corresponding domain W and the local one-parameter action $\theta : W \to M$.

3 Aufgabe

Consider the vector field $X := x^2 \frac{\partial}{\partial x}$ on $M := \mathbb{R}$. Find the local associated flow θ and describe its domain W.

4 Aufgabe

Let $M = GL(2, \mathbb{R})$ and define an action of $\mathbb{R}$ on M by the formula

$$\theta_t(A) := \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix} \cdot A, \quad A \in GL(2, \mathbb{R}),$$

with the dot denotng matrix multiplication. Find the infinitesimal generator.