Berlin Mathematical School

# Topology WS 2006–07

## Homework assignment 11, due 7. Feb. 2007

(1) If $p : Y \to X$ is a covering, then the induced map on fundamental groups is injective. How about the induced map on $H_1$? Prove or give a counter-example.

(2) Prove the strong form of the Five Lemma. (See Bredon IV.5.10 or our discussion on 31.Jan.) That is, show that if $f_2$ and $f_4$ are surjective and $f_5$ is injective, then $f_3$ is surjective. Symmetrically, if $f_2$ and $f_4$ are injective, and $f_1$ is surjective, then $f_3$ is injective.

(3) Suppose that $A$ is a nonempty subset of $X$ and that $A$ is acyclic. (Remember, this means that the reduced homology of $A$ vanishes.) Show $H_p(X, A) \cong \tilde{H}_p(X)$.

(4) Prove from the Eilenberg–Steenrod axioms that for any space $X$ we have $H_p(X, X) = 0$ for all $p$.