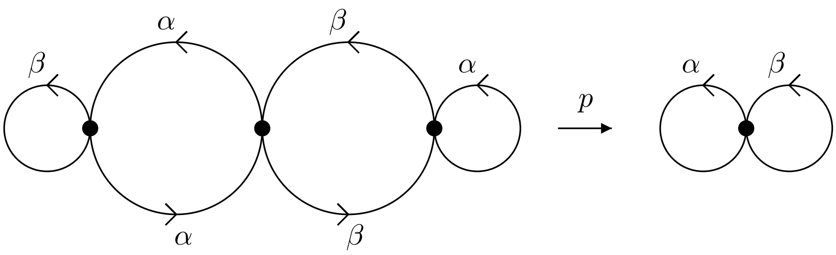
Berlin Mathematical School

Topology WS 2006–07

Homework assignment 9, due 17. Jan. 2007

(1) Suppose $p : (Y, y_0) \to (X, x_0)$ is a covering, and suppose α and β are two paths in X from x_0 to some point x_1. Let $\tilde{\alpha}$ and $\tilde{\beta}$ be their lifts to paths in Y starting at y_0. Show that $\tilde{\alpha}$ and $\tilde{\beta}$ have the same endpoint if and only if $[\alpha * \bar{\beta}] \in \pi_1(X, x_0)$ is in $p_*\big(\pi_1(Y, y_0)\big)$

(2) Recall the covering space from assignment 6:



What is its group of deck transformations? Is this covering regular?

(3) If $\pi_1(X, x_0)$ is abelian, show that every covering of X is regular.

(4) Remember that an *abelian cover* of X is a G-cover for some abelian group G. Show that any connected abelian cover of X has the form $\hat{X}/H$ where $\hat{X}$ is the universal abelian cover and $H < H_1X$ is some subgroup of the first homology group of X.