Berlin Mathematical School

Topology WS 2006–07

Homework assignment 3, due 15. Nov. 2006

(1) We say that a space X has the *fixed-point property* if every map $f : X \to X$ has a fixed point. (The Brouwer Fixed-Point Theorem then says that the closed n-disk has the fixed-point property.) Recall that if Y is a subspace of X, then a *retraction* $f : X \to Y$ is a continuous map which restricts to the identity on Y; we say that Y is a *retract* of X. Show that if X has the fixed-point property and Y is a retract of X, then Y also has the fixed-point property.

(2) Suppose $A \subset \mathbb{R}^2$ is connected and closed, and $P \in A$. Show that $[\omega_P] = 0 \in H^1(\mathbb{R}^2 \setminus A)$ if and only if A is unbounded.

(3) Suppose U and V are connected, open subsets of $\mathbb{R}^2$. Show that if $H^1(U \cup V) = 0$ then $U \cap V$ is also connected. (Hint: use the proposition about the kernel of the coboundary map δ.)

(4) Suppose $X \subset \mathbb{R}^2$ is homeomorphic to a figure-eight 8, that is to two circles sharing one point. Show that $\mathbb{R}^2 \setminus X$ has exactly three components.