Berlin Mathematical School

Topology WS 2006–07

Homework assignment 2, due 8. Nov. 2006

For the first two problems, let $U := \mathbb{R}^2 \smallsetminus \{0\}$.

(1) Suppose γ_0 and γ_1 are two paths from P to Q in U. Show that if $W(\gamma_0, 0) = W(\gamma_1, 0)$ then the paths are homotopic in U with fixed endpoints. (Hint: express the homotopy in polar coordinates.)

(2) If $f : \mathbb{S}^1 \to U$ has $W(f, 0) = 0$, show that f can be extended to a continuous map $\bar{f} : D^2 \to U$. (Hint: use the first problem to show the path f is homotopic to a constant path.)

(3) Suppose $f : \mathbb{S}^1 \to \mathbb{S}^1$ has no fixed point. Show that $\deg(f) = 1$, and thus that f is surjective.

(4) Suppose $f : \mathbb{S}^1 \to \mathbb{S}^1$ is *even* in the sense that $f(-p) = f(p)$. Show that $\deg(f)$ is even.

(5) Suppose $f : \mathbb{S}^2 \to \mathbb{S}^2$ never has the same value at two antipodal points (that is, $f(-p) \neq f(p)$ for all p). Show that f is surjective. (Hint: $\mathbb{S}^2 \smallsetminus \{x\}$ is homeomorphic to $\mathbb{R}^2$.)