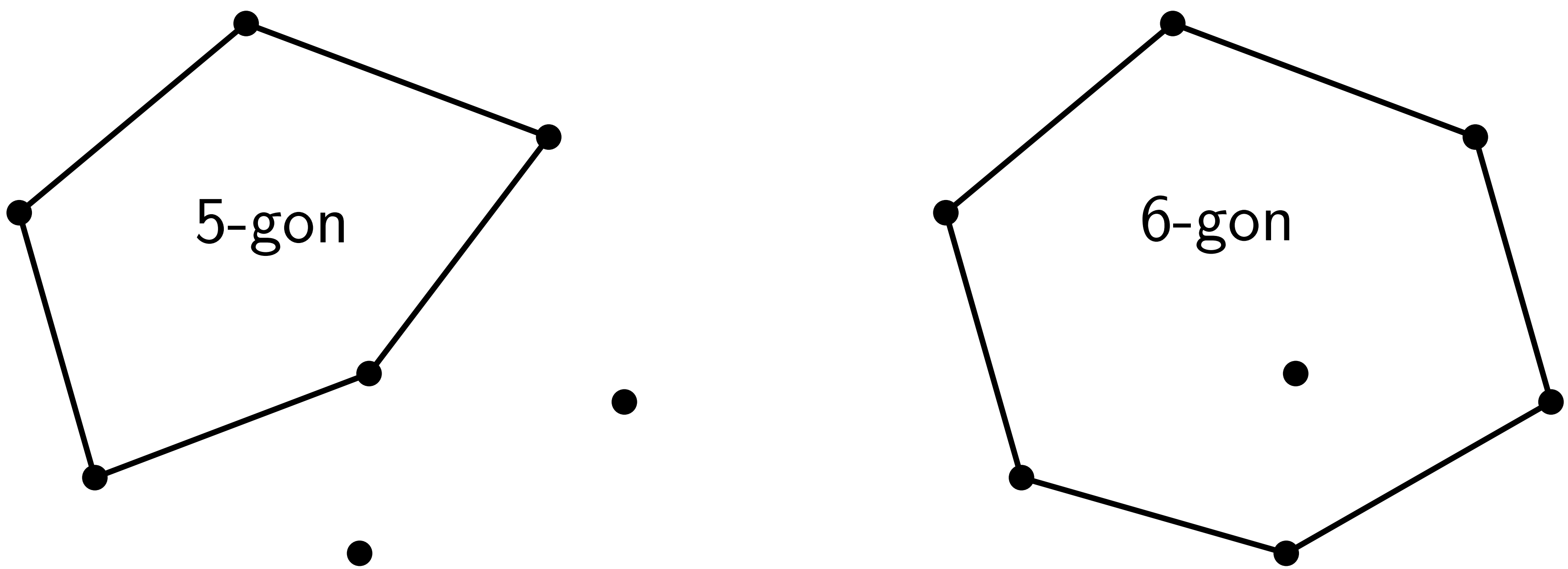
Holes in convex drawings

Helena Bergold, Manfred Scheucher and Felix Schröder

k-Gons

a k-gon in a point set S is a convex polygon spanned by k points of S

Theorem (Erdős & Szekeres 1935).
$\forall\ k \in \mathbb{N}$, $\exists$ a smallest integer $g(k)$ such that every set of $g(k)$ points determines a k-gon.



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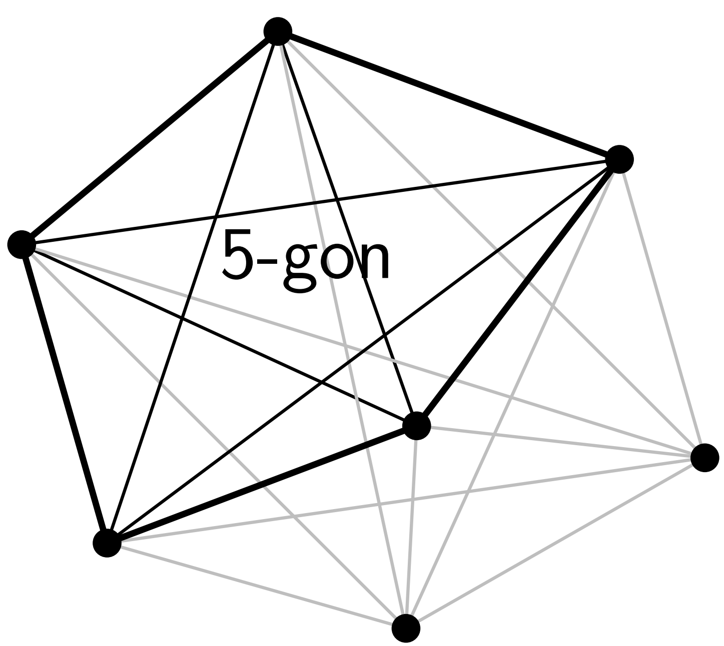

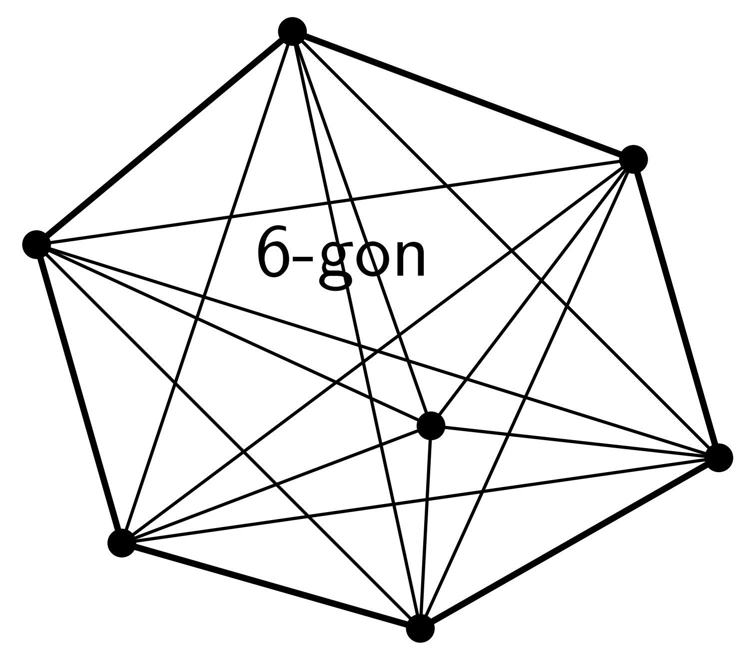

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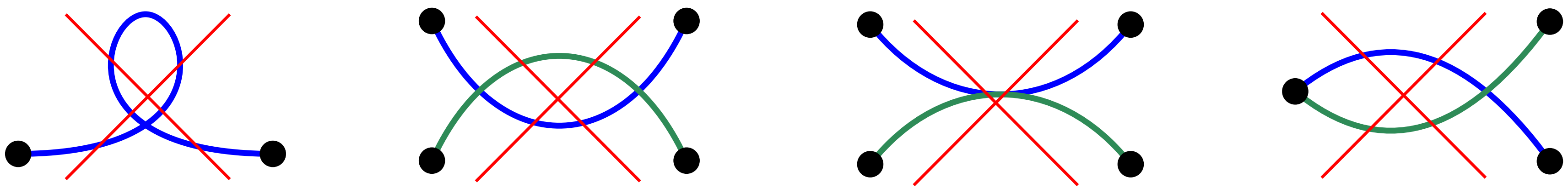
- $g(k) = 2^{k+o(k)}$ [Suk '16]
- applies to pseudolinear drawings [Holmsen, Mojarrad, Pach and Tardos '17]
- $g(k) = 2^{k-2} + 1$ conjectured

Simple Drawings

- edges are Jordan arcs (no self-intersections)
- any two edges intersect in at most one point (common vertex or proper crossing)



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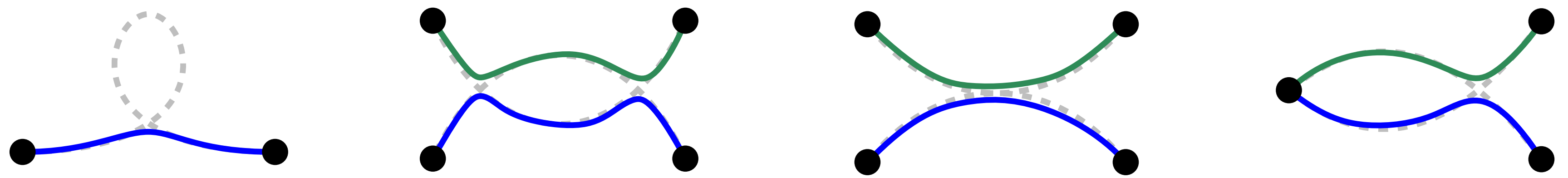

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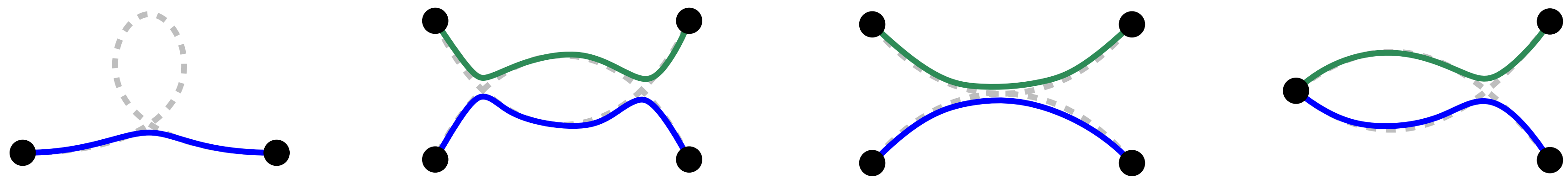
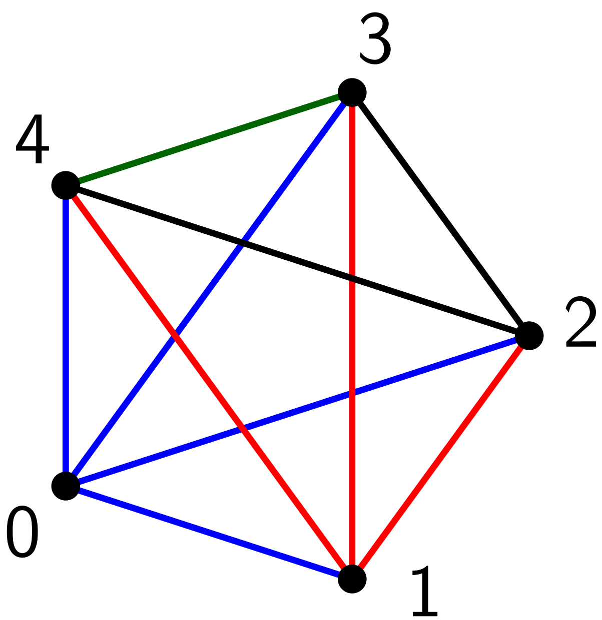
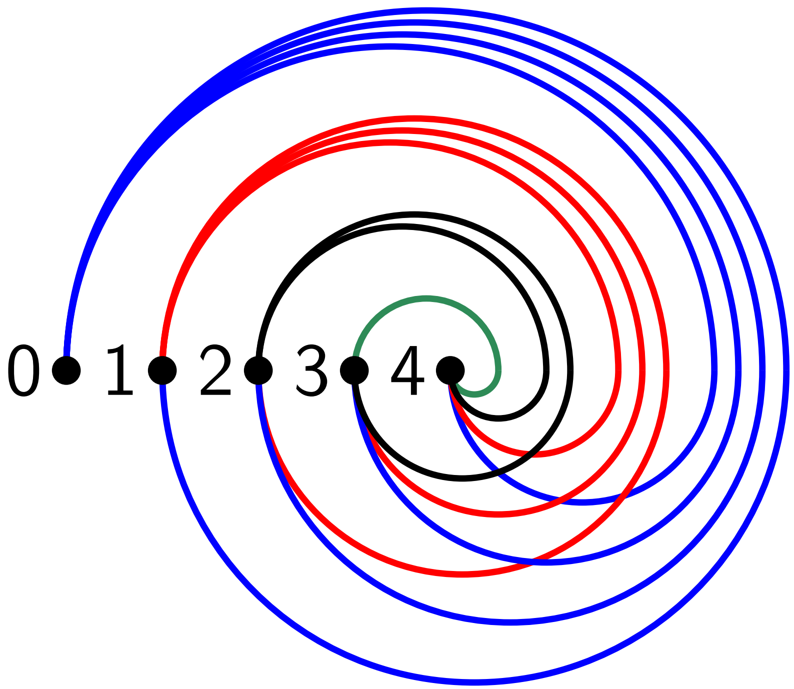


- generalize crossing-minimal drawings
- and geometric drawings

k-Gons in Simple Drawings

[Pach, Solymosi & Tóth '03]: $\forall$ simple drawing of $K_{\widetilde{R}(k,\ell)}$
$\exists$ *convex* C_k (k-gon) or *twisted* T_ℓ subdrawing





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- $\widetilde{R}(k,\ell) \leq c^{(k\ell)^4}$ [Pach, Solymosi & Tóth '03]
- $\widetilde{R}(k,\ell) \leq c^{(k\ell)^2 \log(k) \log(\ell)}$ [Suk & Zeng '22]

Variant: k-Holes

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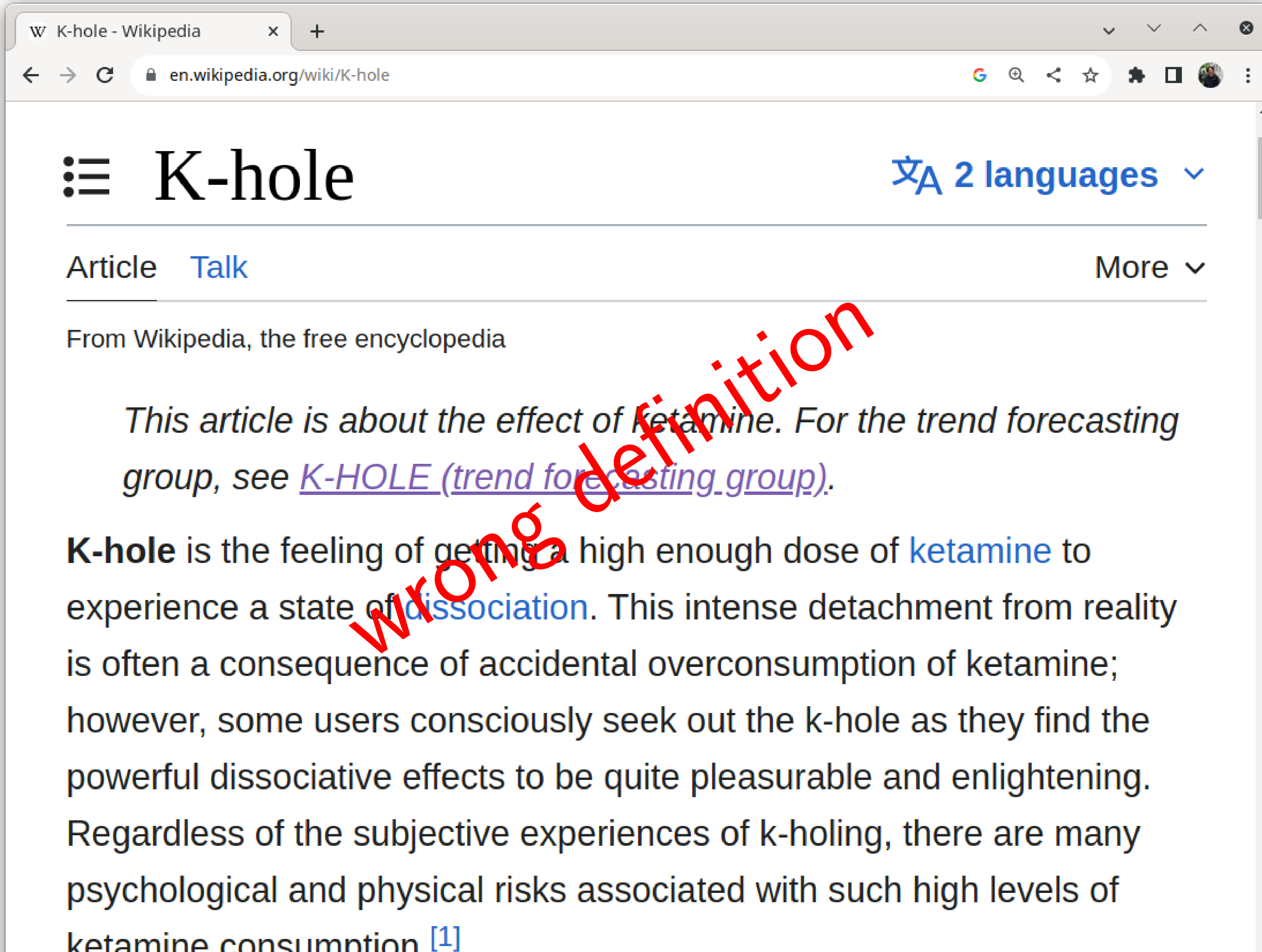

K-hole - Wikipedia

en.wikipedia.org/wiki/K-hole

K-hole

2 languages

Article Talk More

From Wikipedia, the free encyclopedia

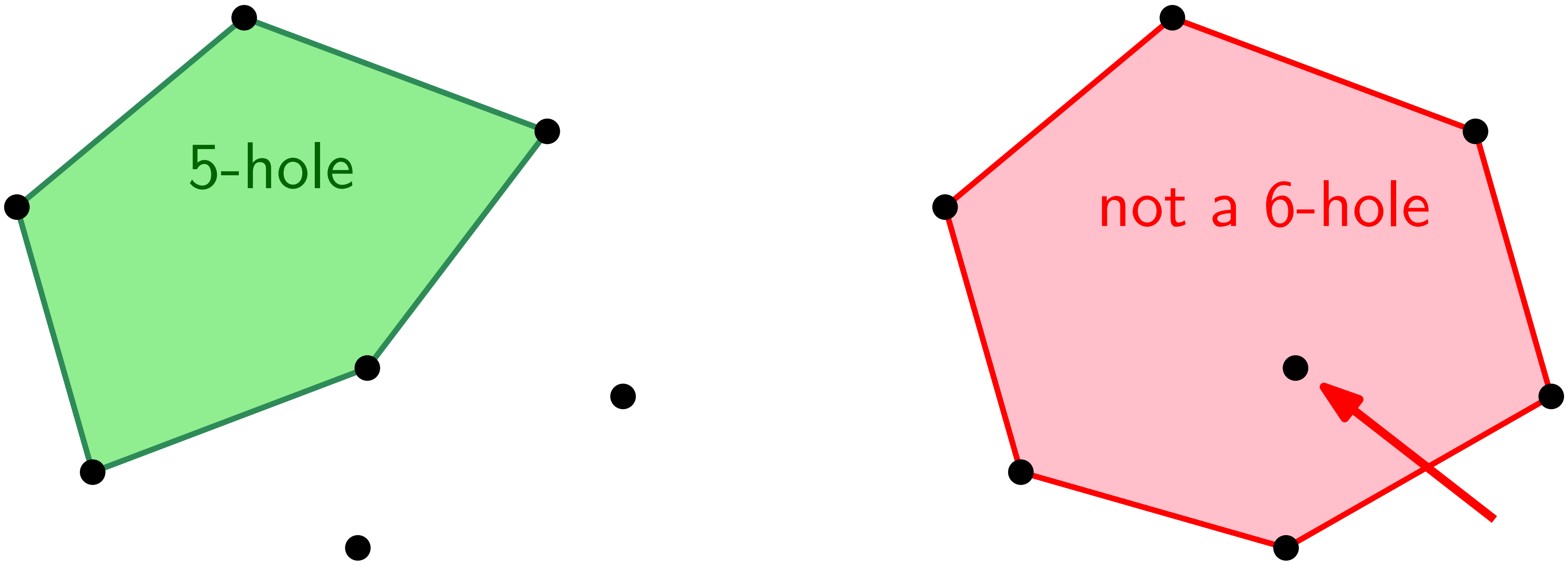
This article is about the effect of ketamine. For the trend forecasting group, see K-HOLE (trend forecasting group).

K-hole is the feeling of getting a high enough dose of ketamine to experience a state of dissociation. This intense detachment from reality is often a consequence of accidental overconsumption of ketamine; however, some users consciously seek out the k-hole as they find the powerful dissociative effects to be quite pleasurable and enlightening. Regardless of the subjective experiences of k-holing, there are many psychological and physical risks associated with such high levels of ketamine consumption.[1]

Variant: k-Holes

Erdős, 1970's: For k fixed, does every sufficiently large point set determine a k-hole?

a k-hole in a point set S is a k-gon which contains no other points of S



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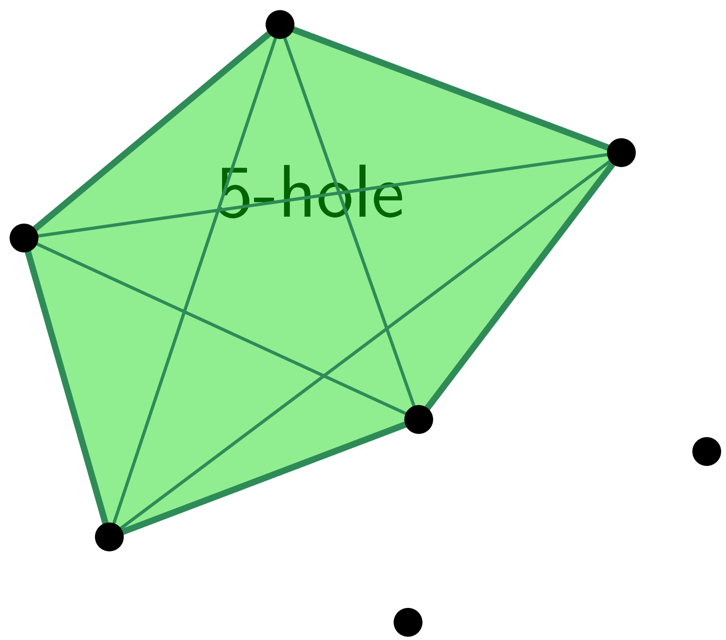

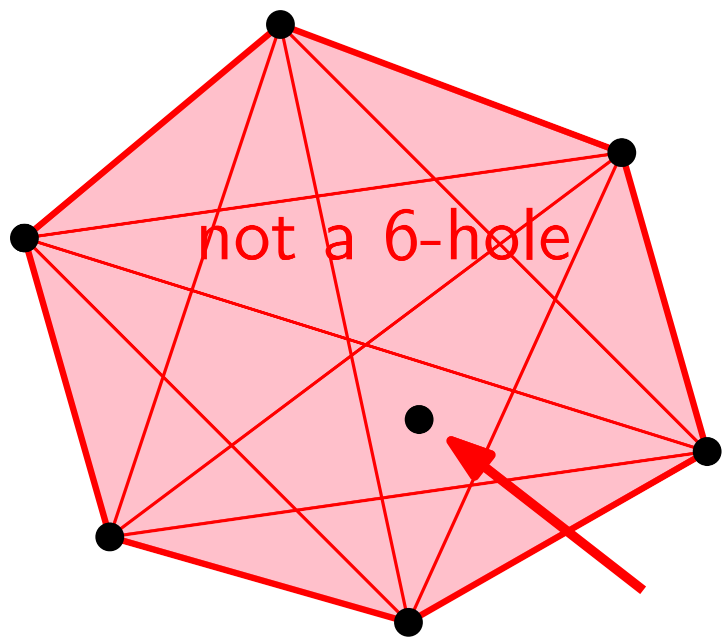

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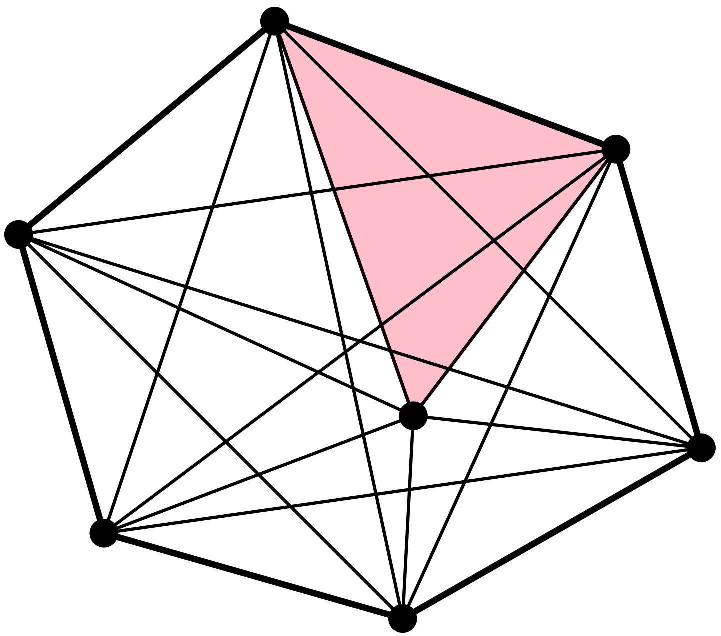
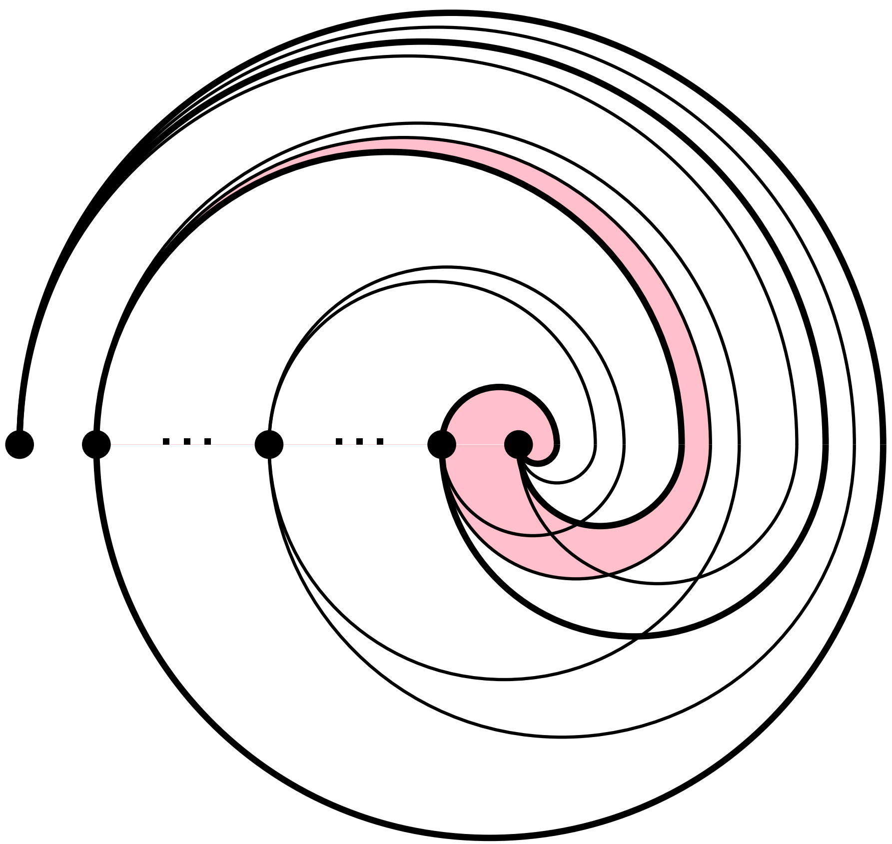
- Sufficiently large point sets $\Rightarrow$ $\exists$ 6-hole [Gerken '06; Nicolás '07]
- $\exists$ arbitrarily large point sets with no 7-hole [Horton '83]

k-Holes in Simple Drawings

a k-hole in a simple drawing of K_n is a C_k such that every triangle has an empty side





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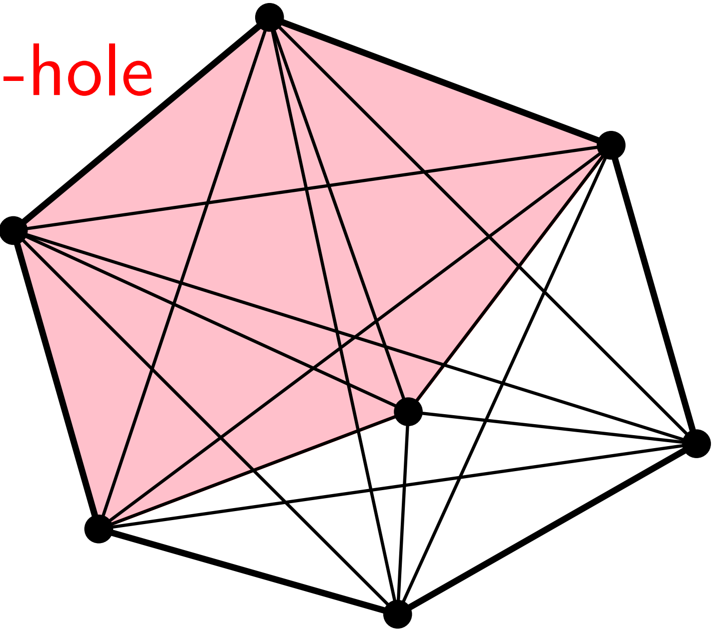

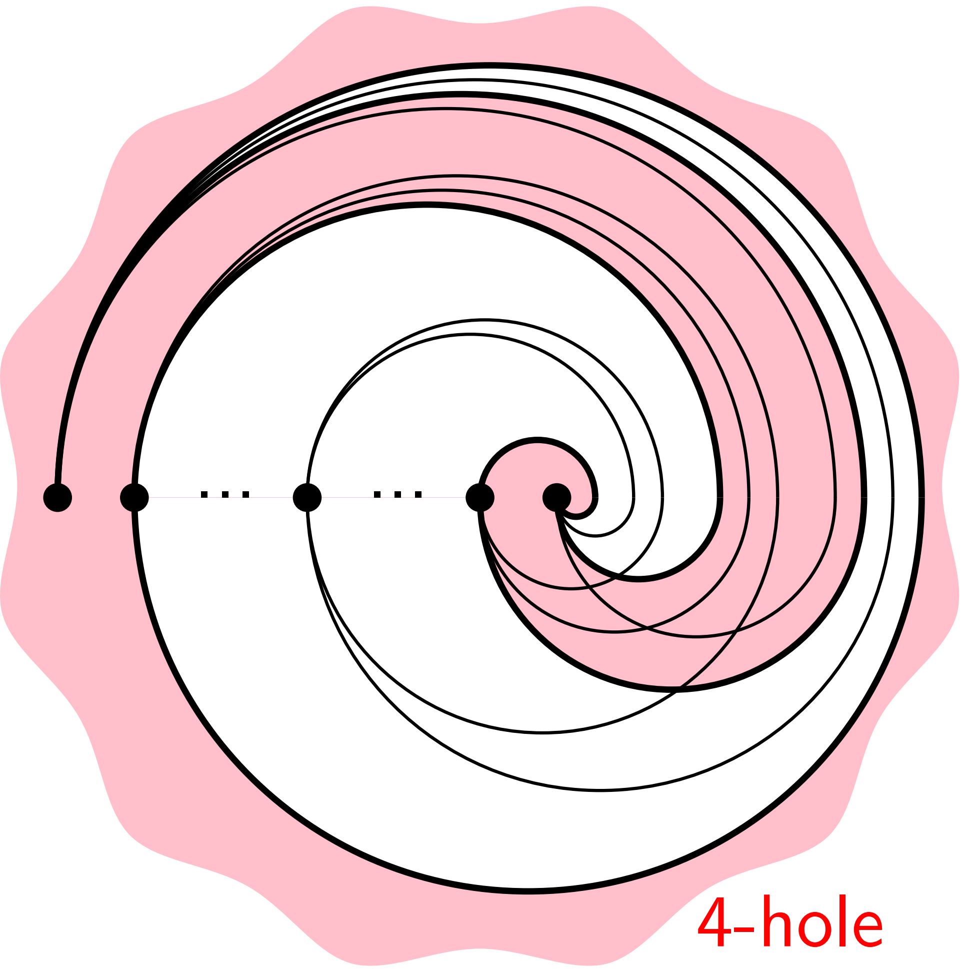

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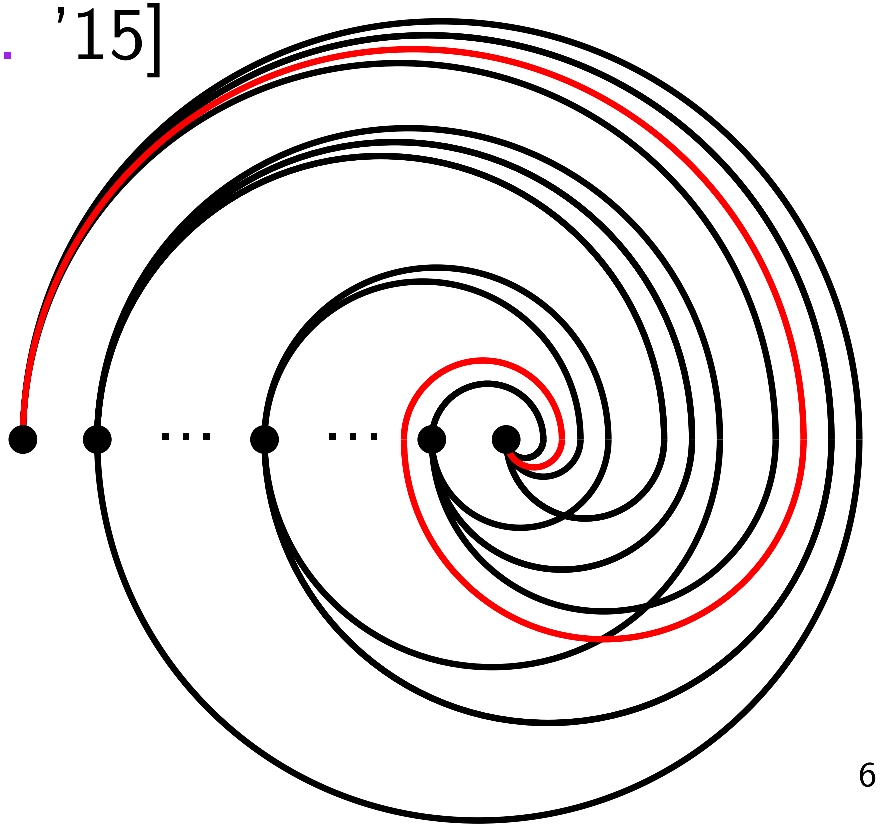

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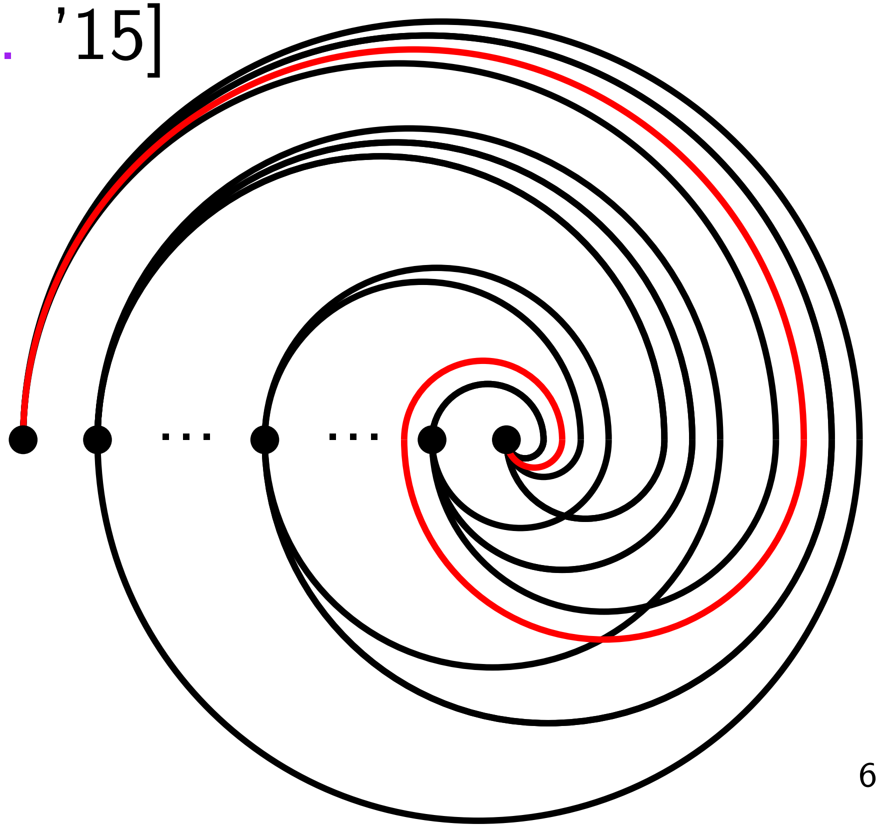
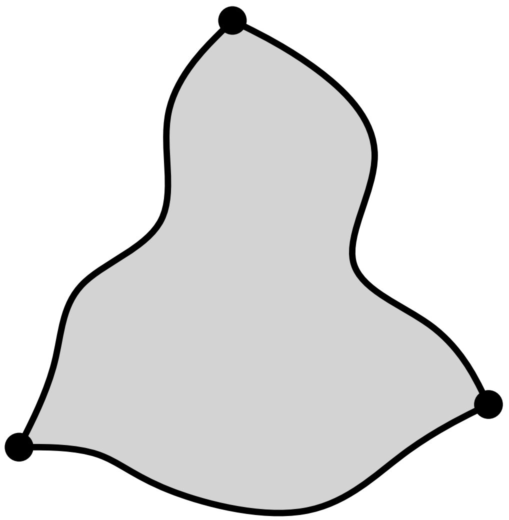


Intermediate: Convex Drawings

In a simple drawing of K_n any 3 vertices induce a triangle $\triangle$ with a bounded side and an unbounded side



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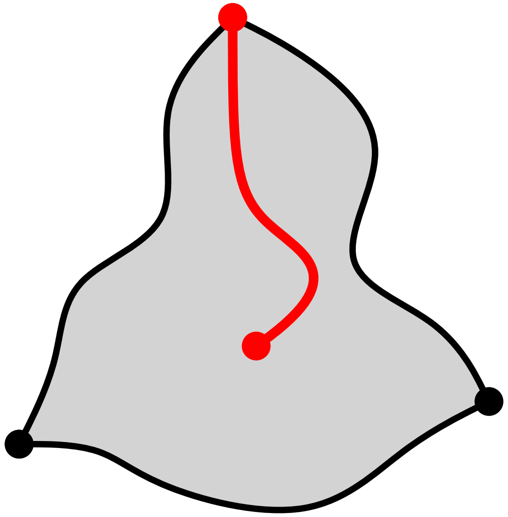
Definition (Arroyo et al. 2017).
A simple drawing is convex iff $\forall$ triangle $\exists$ convex side S,

i.e., $\forall$ vertices a, b from S
the edge ab is fully contained in S



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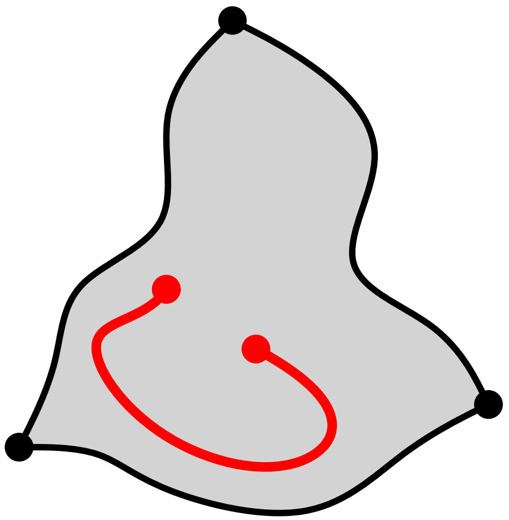

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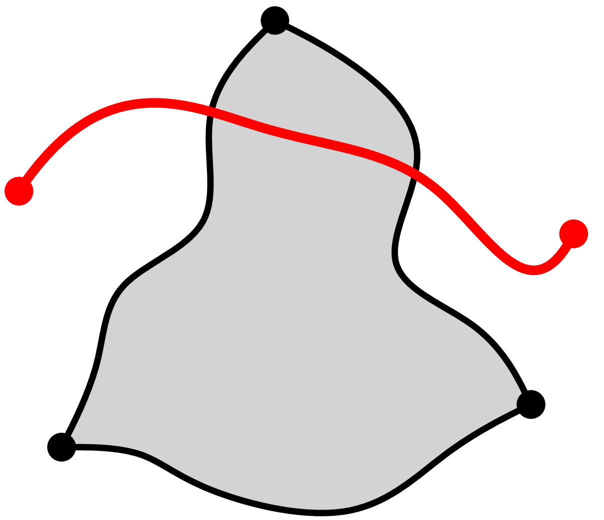

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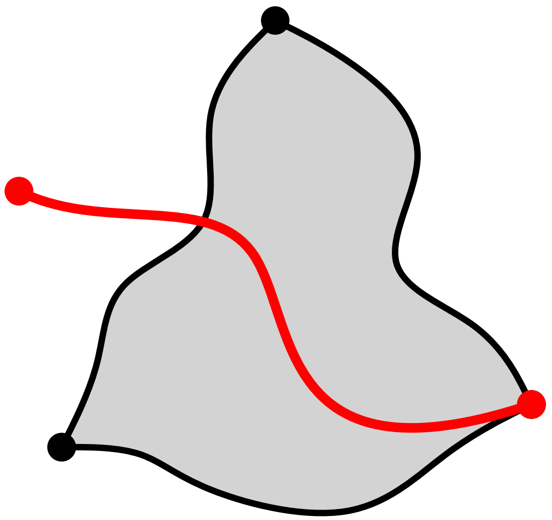


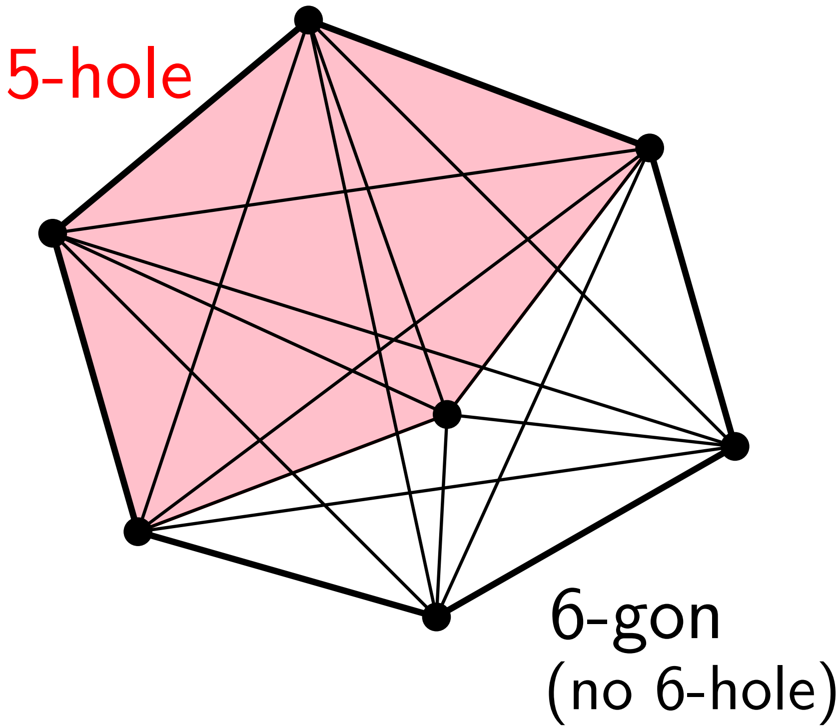
Convexity Hierarchy (Arroyo et al. '17)

- geometric
  (order types, realizable acyclic rank 3 OM)
- pseudolinear / f-convex
  (abstract order types, acyclic rank 3 OM, CC-system)
- h-convex
- convex
- simple

Holes Revised

$k \geq 4$: k-hole $\Leftrightarrow$ convex side of C_k is empty

convex side of C_k := union of convex sides of triangles in induced subdrawing $\mathcal{D}[C_k]$



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- **Theorem:** n sufficiently large $\Rightarrow$ 6-holes exist

proof: 1. find large pseudolinear drawing
2. empty hexagon theorem for pseudolinear [S.'23]

- **Theorem:** C_k minimal k-gon with $k \geq 5$
$\Rightarrow$ the convex side of C_k induces pseudolinear drawing

Discussion

- 5-holes in convex drawings of $K_{n\geq 13}$?

- largest pseudolinear subdrawing in convex drawing?

- largest C_k in convex drawing?

  $\widetilde{R}_{conv}(k) \leq \widetilde{R}(k,5) \leq c^{k^2 \log(k)}$ via [Suk & Zeng '22]

  (convex drawings do not contain twisted T_5)


THANK YOU
+1