Blocking Delaunay Triangulations from the Exterior

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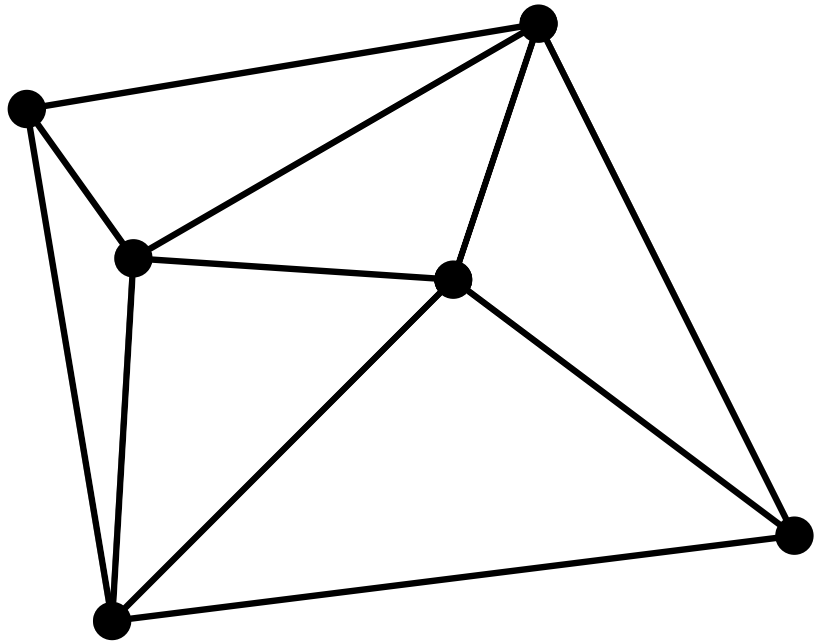
Delaunay Triangulation

- pq edge in $DT(P)$ iff $\exists$ empty circle trough p and q



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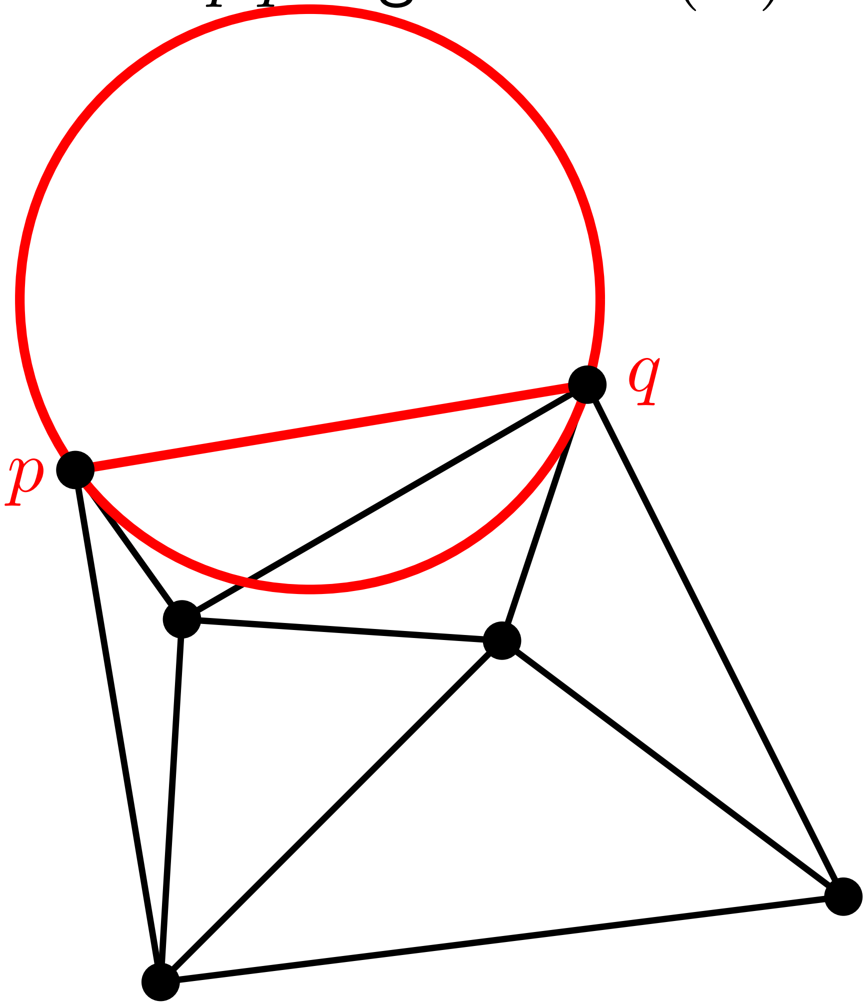

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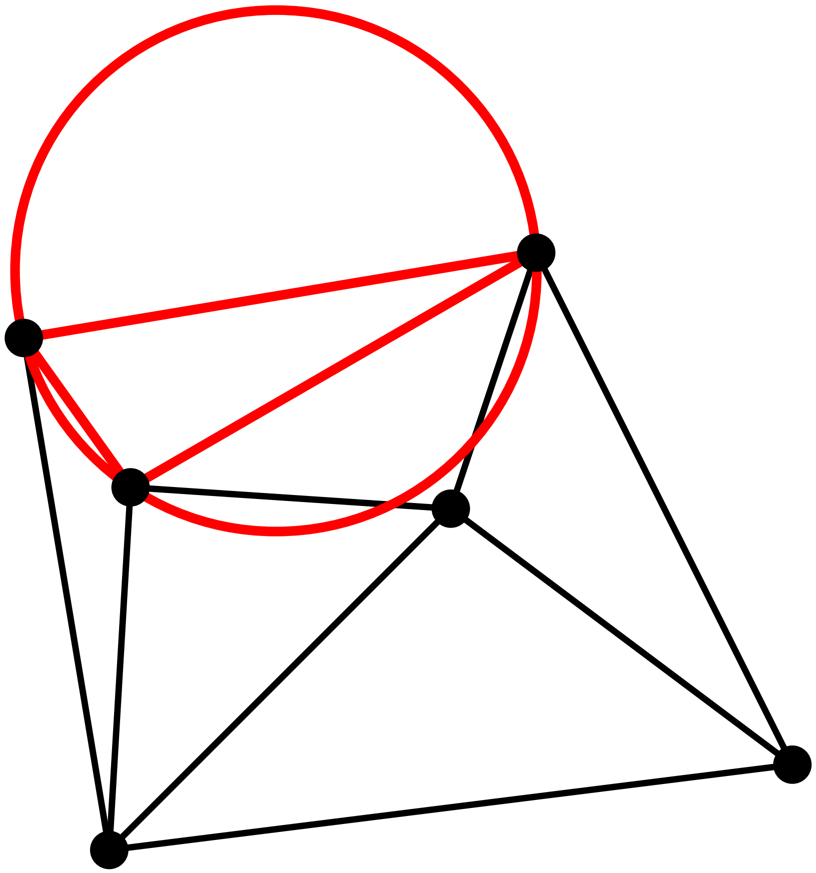

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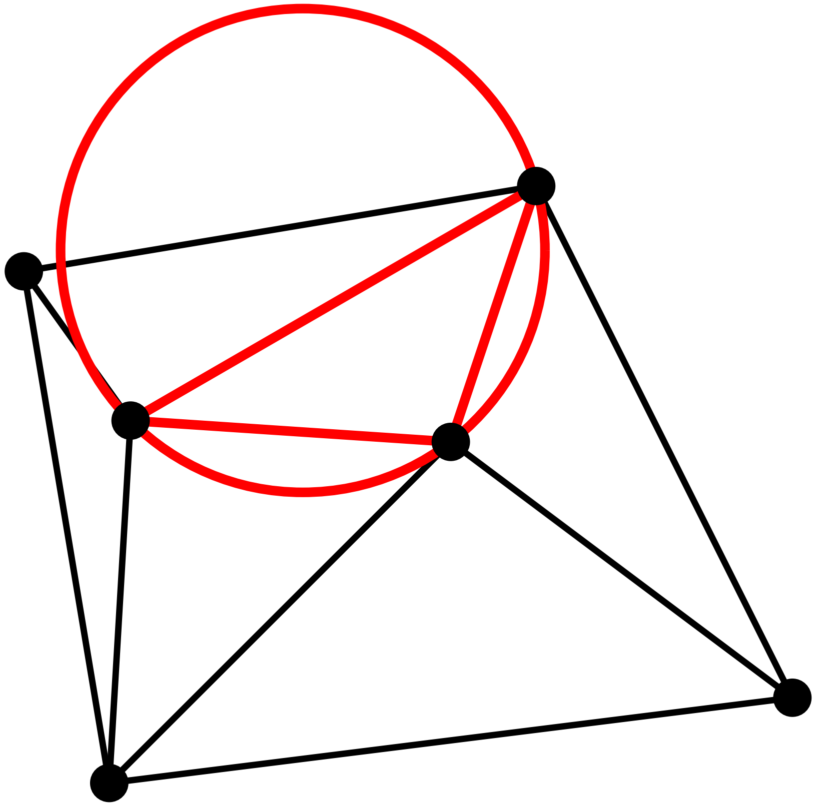

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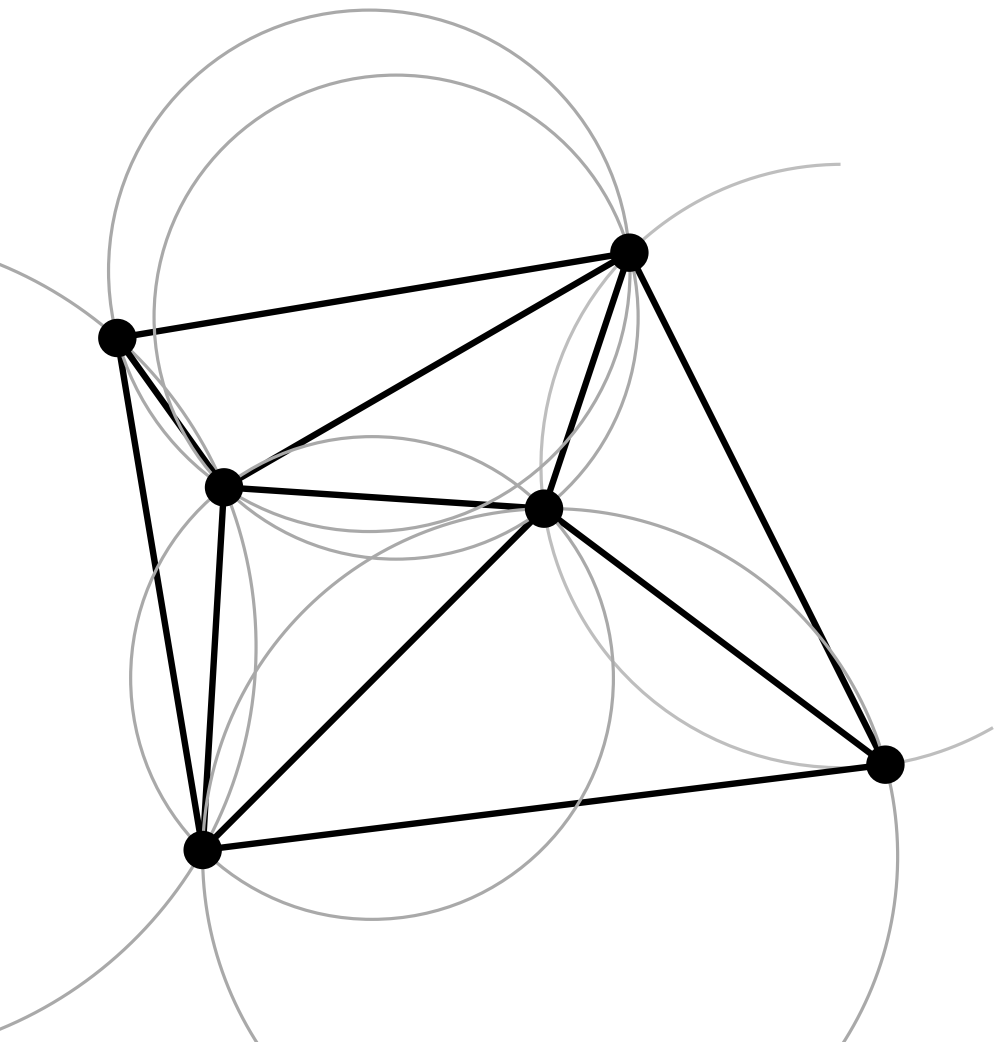

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  - dual *Voronoi* diagram:
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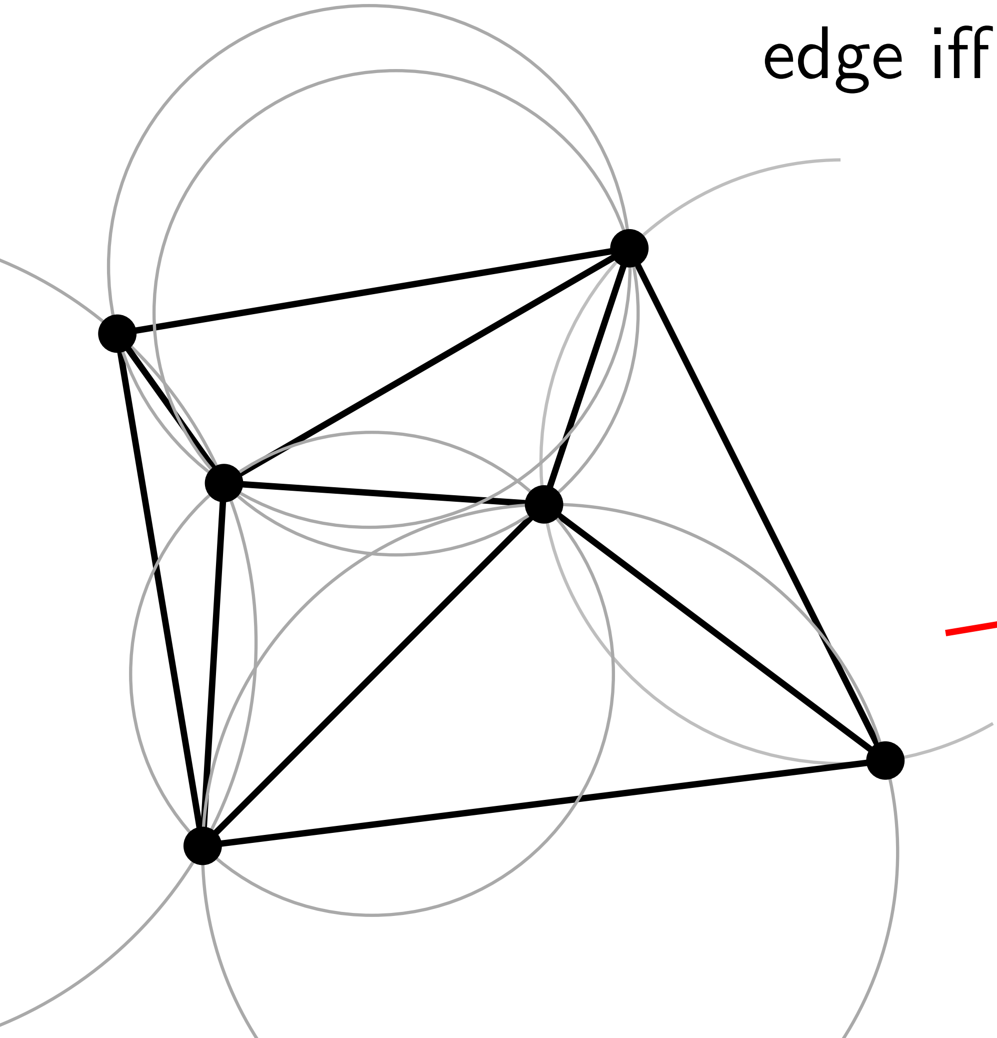

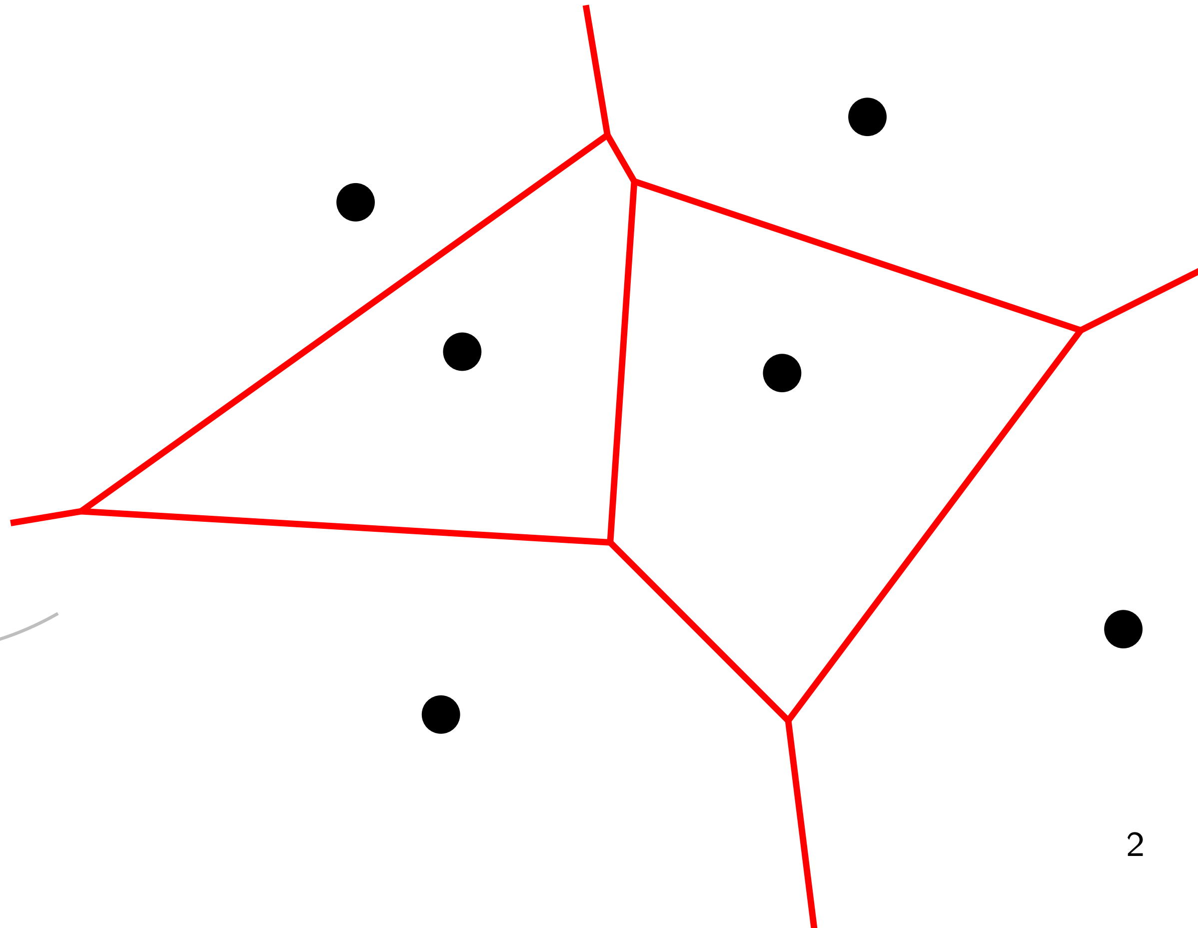

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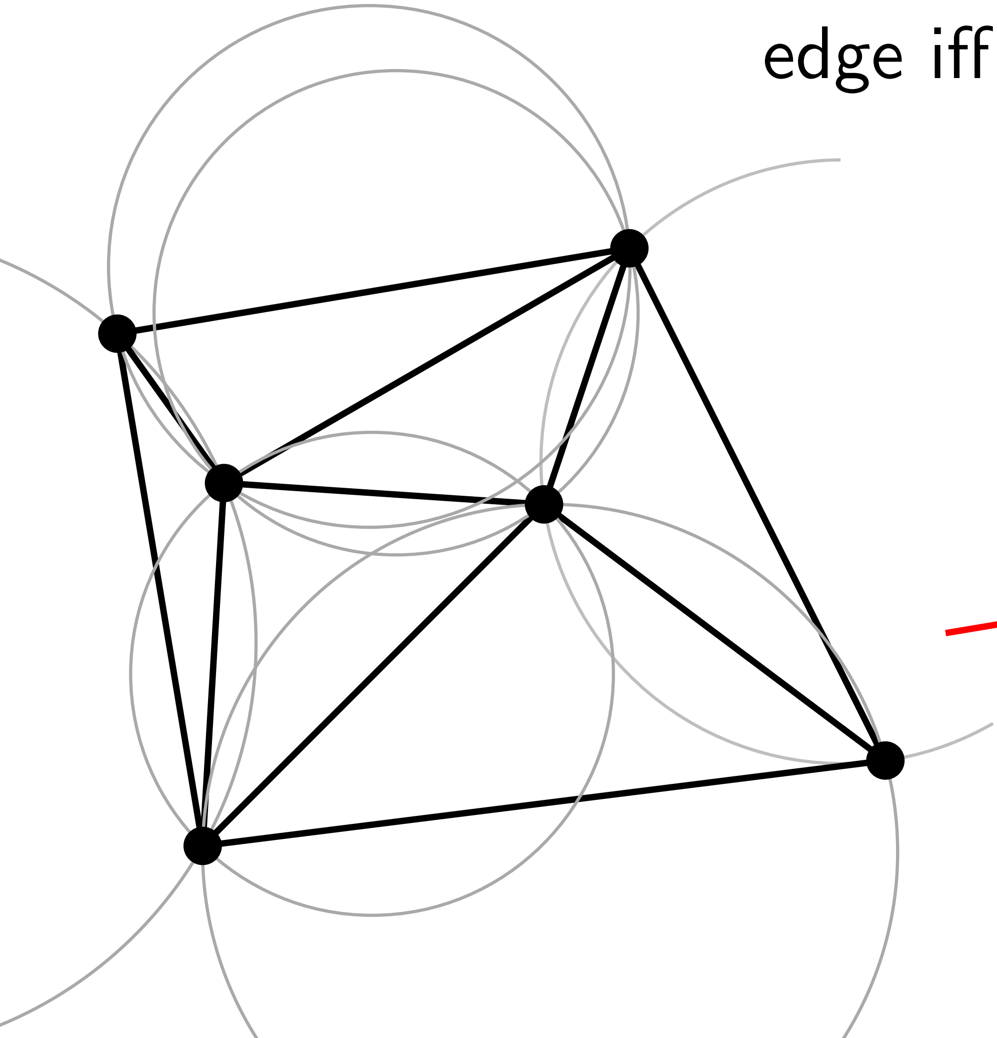

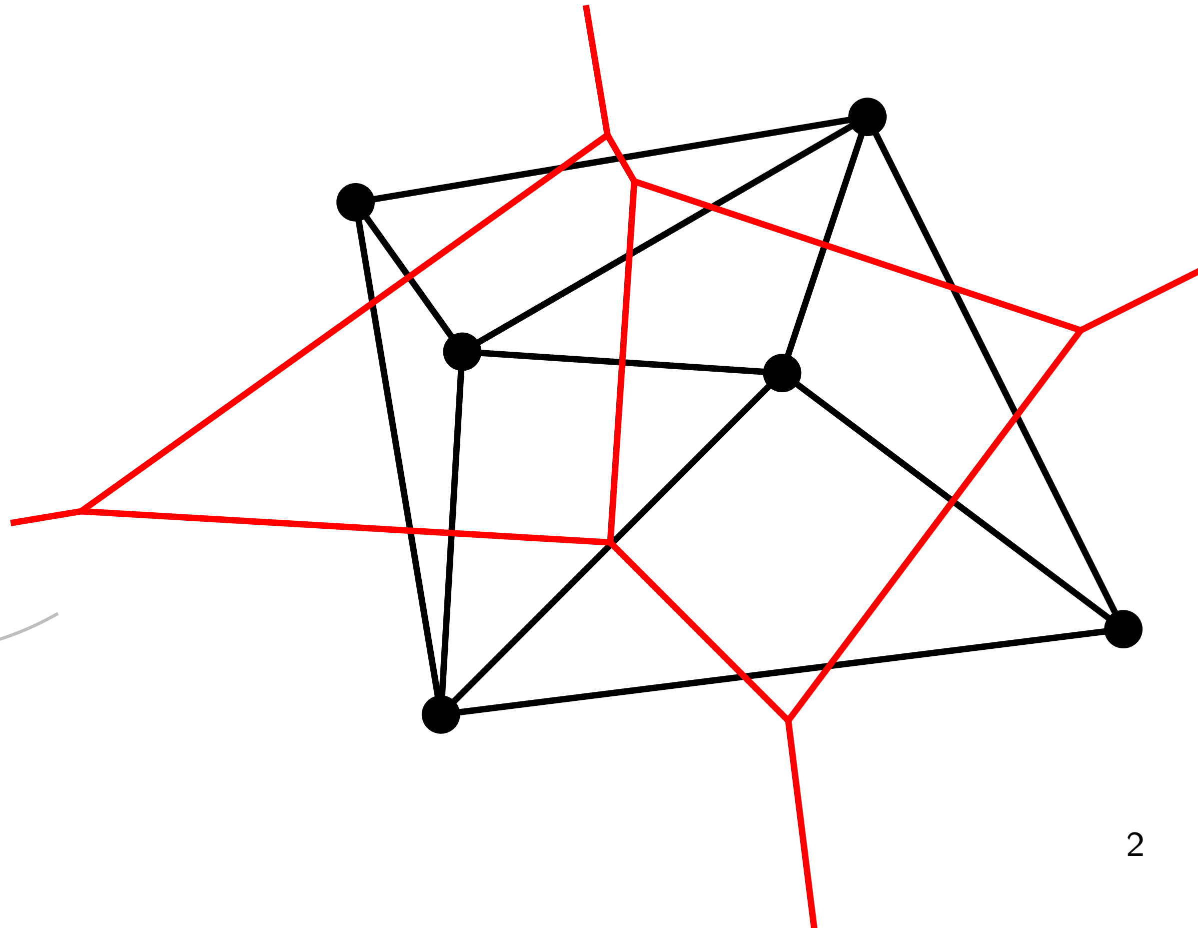

Blocking a DT

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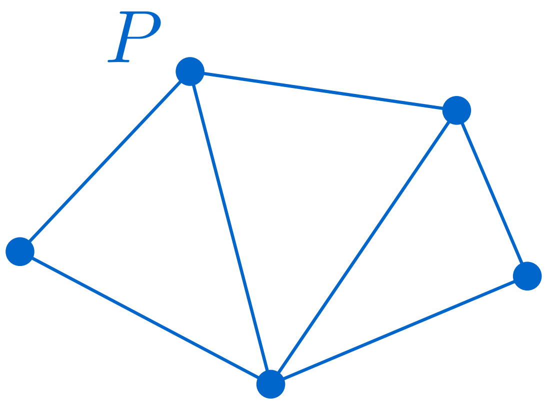
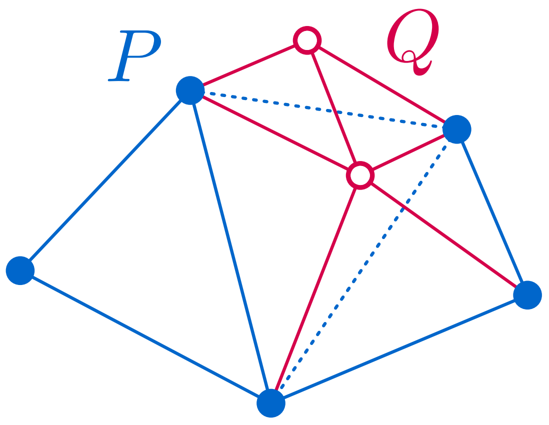
  :⇔ $p_1 p_2$ is not an edge of the DT of $P \cup Q$





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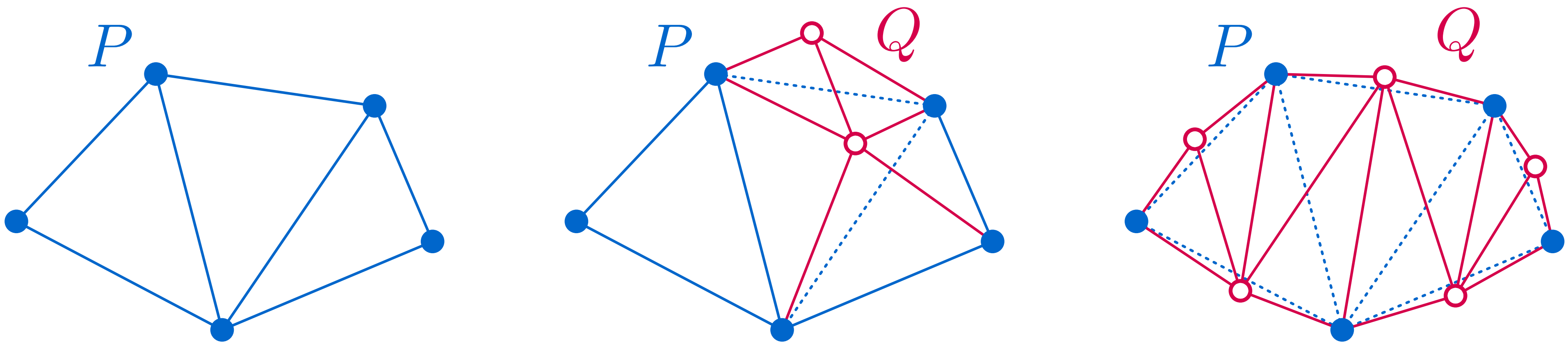

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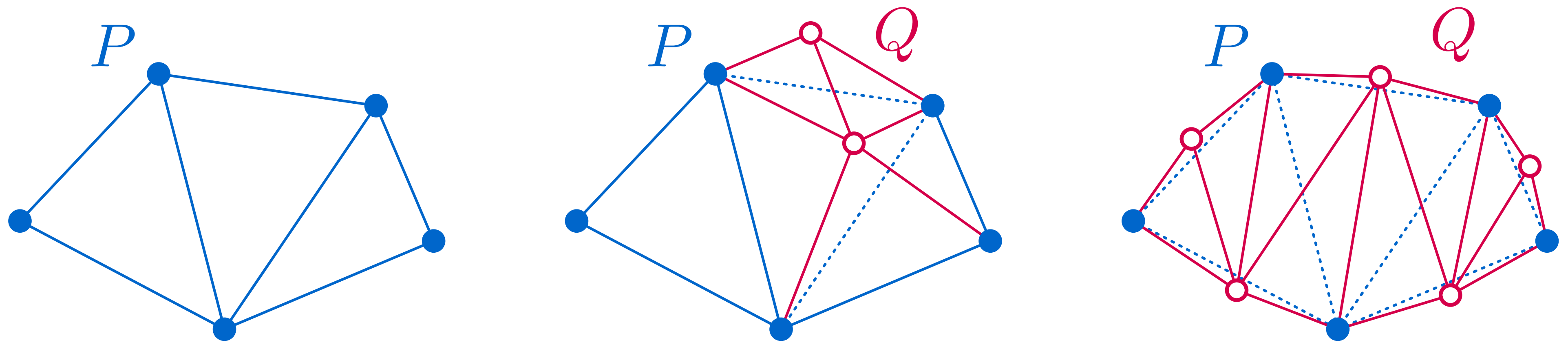

- Q *blocks* P if all edges spanned by P are blocked
- Moreover, Q *blocks* P *from the exterior* if all points of Q lie outside $\mathrm{conv}(P)$

Literature

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Biniaz 2021: n points always necessary

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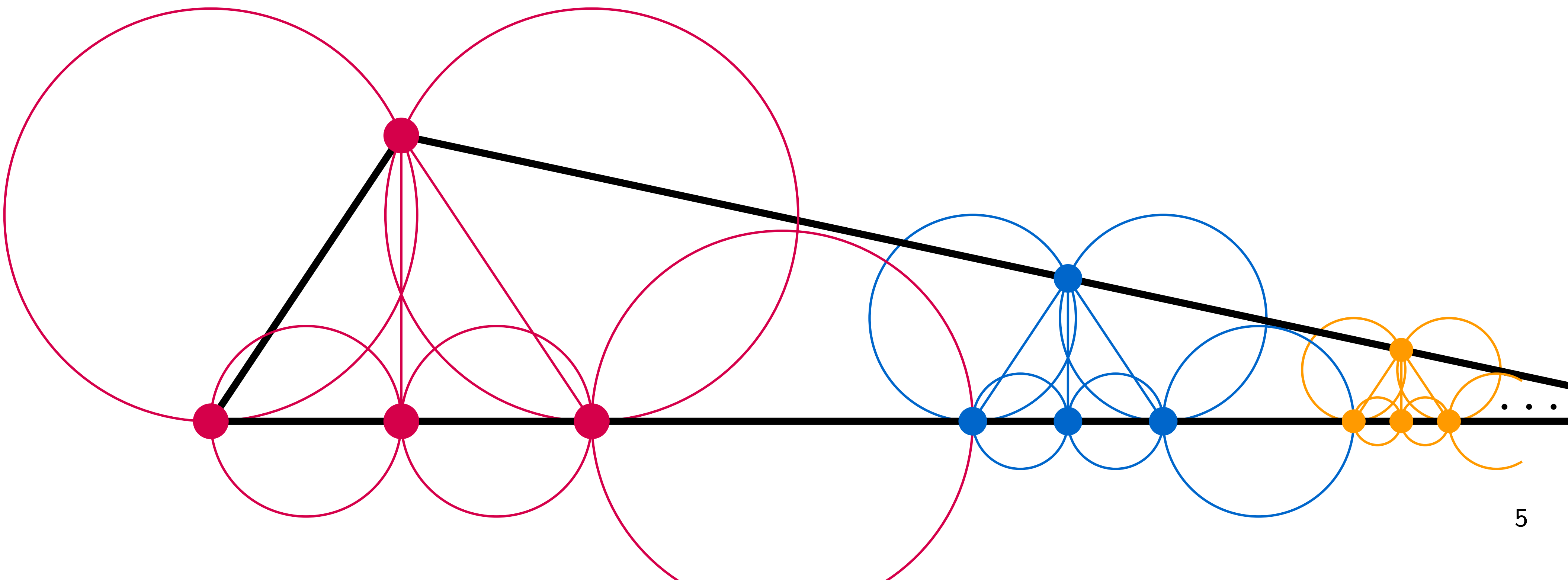
Theorem 1: For $k \in \mathbb{N}$, $\exists$ set P of $4k$ points in general position that requires $5k - 5$ exterior-blocking points.



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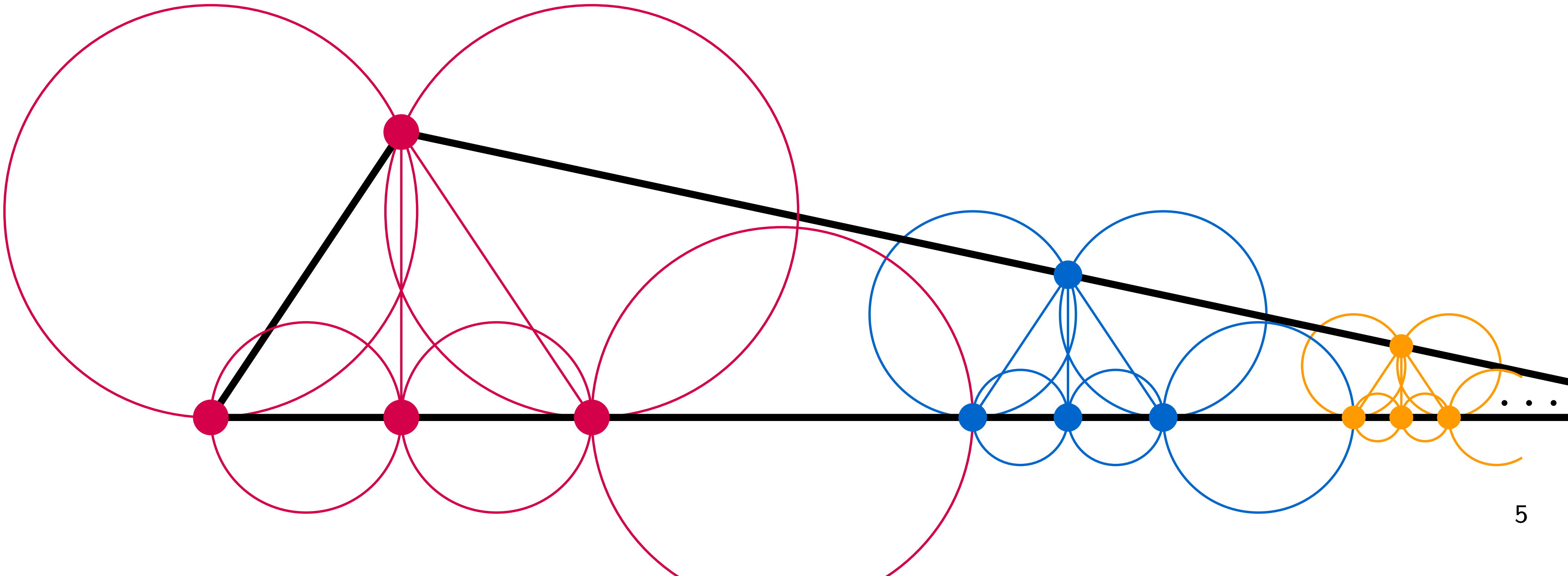
$\Rightarrow$ minimal blocking sets of certain point sets must contain inner points



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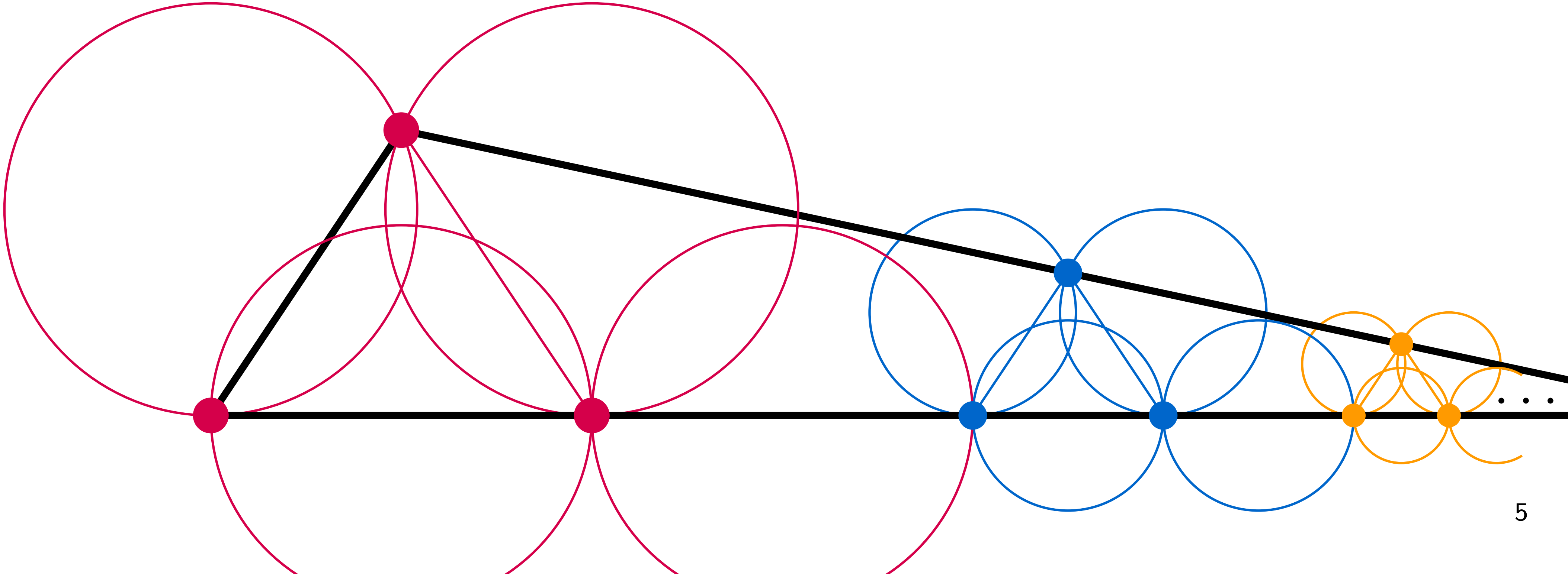
Theorem 2: For $k \in \mathbb{N}$, $\exists$ set P of $3k$ points (degenerate) that requires $4k - 2$ exterior-blocking points.



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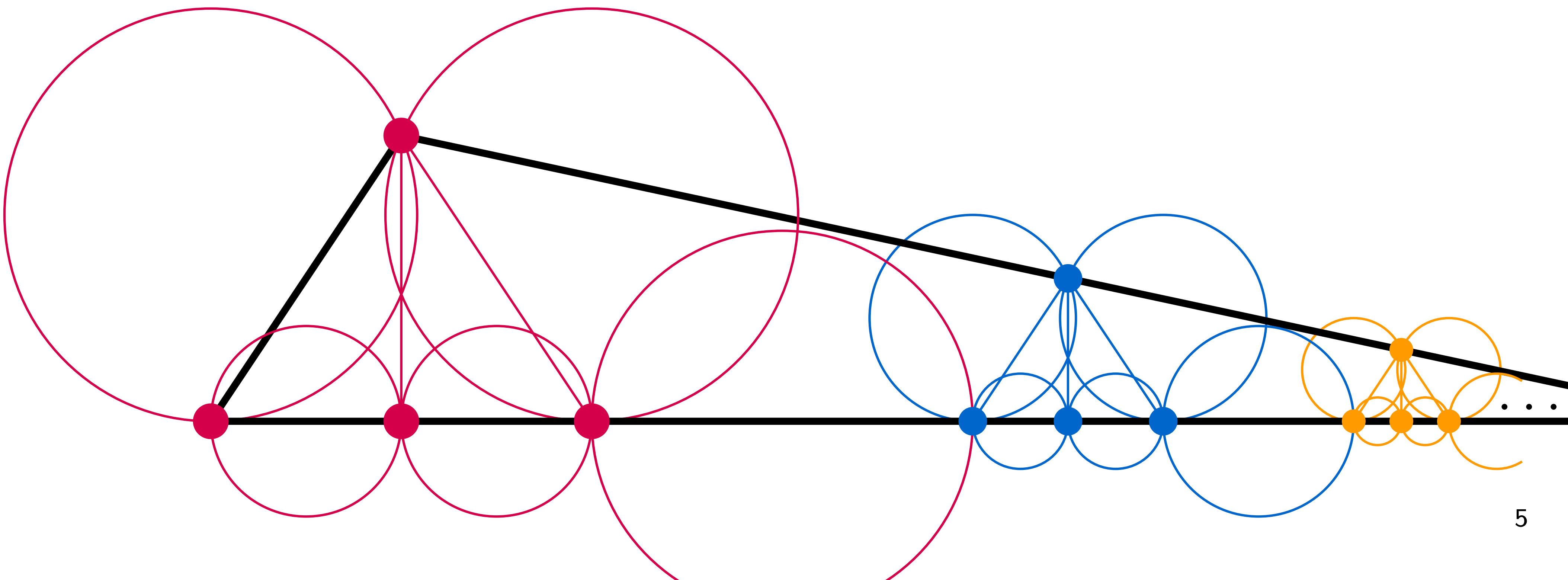

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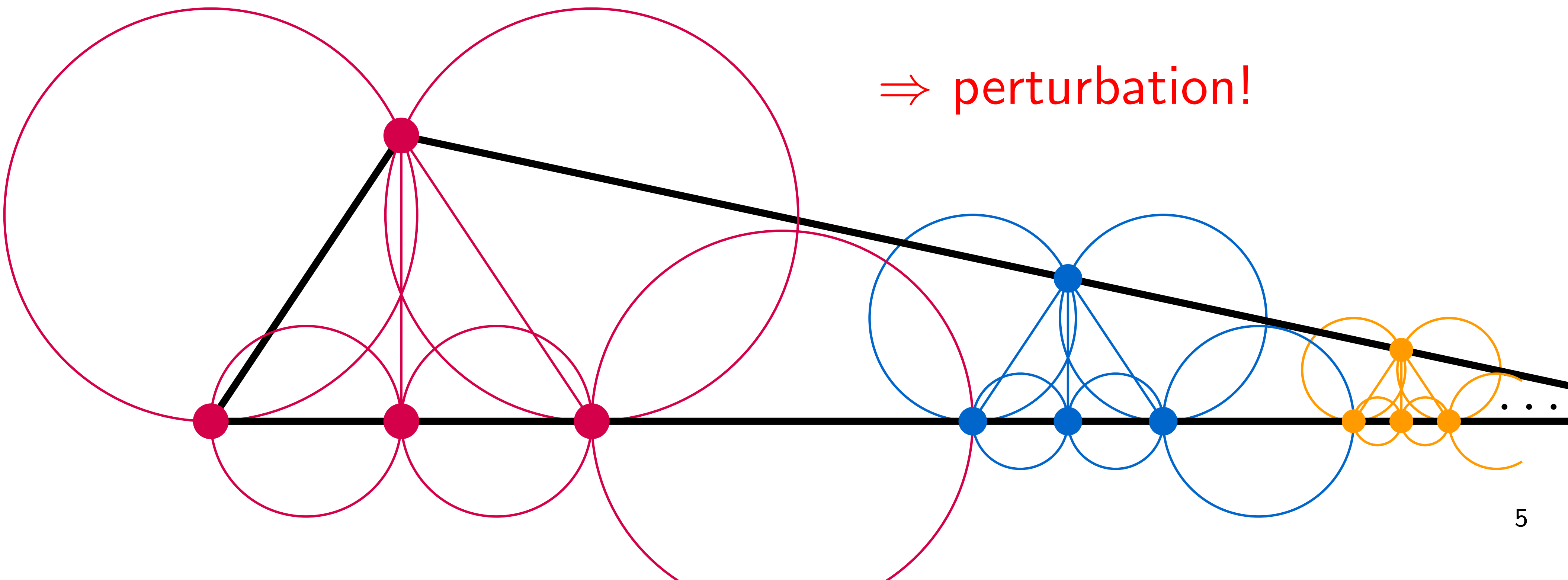

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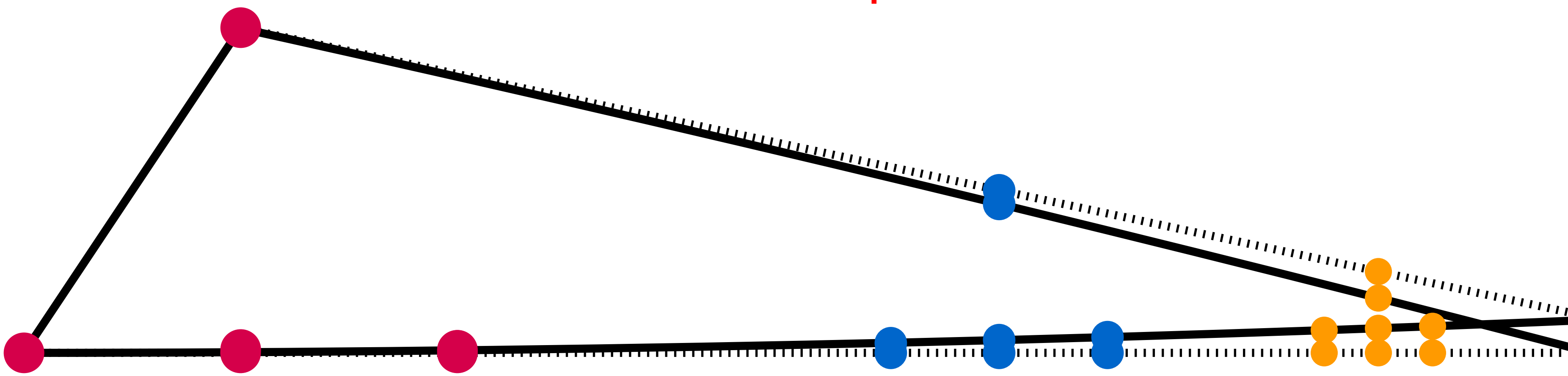
$\Rightarrow$ perturbation!



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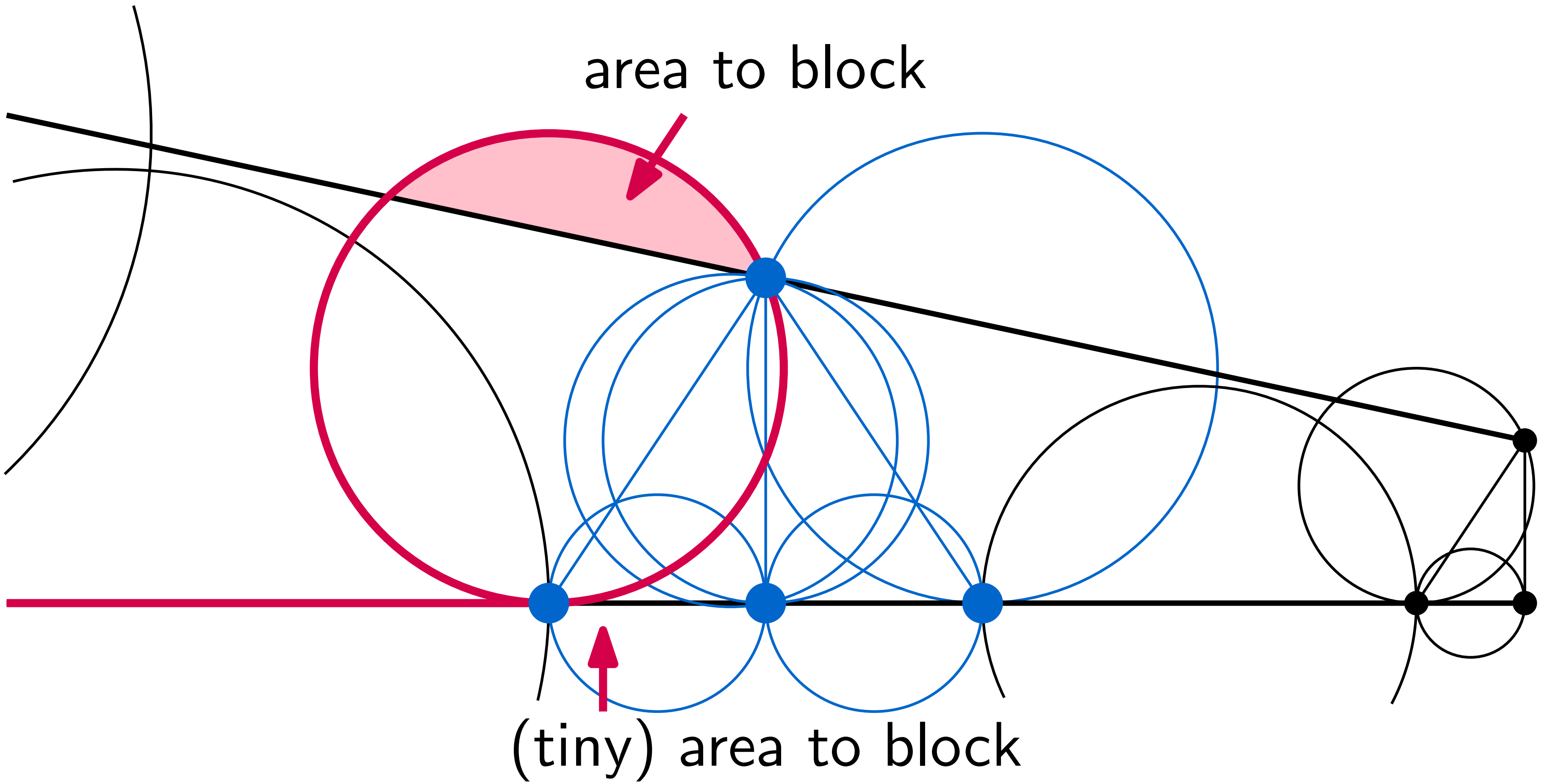

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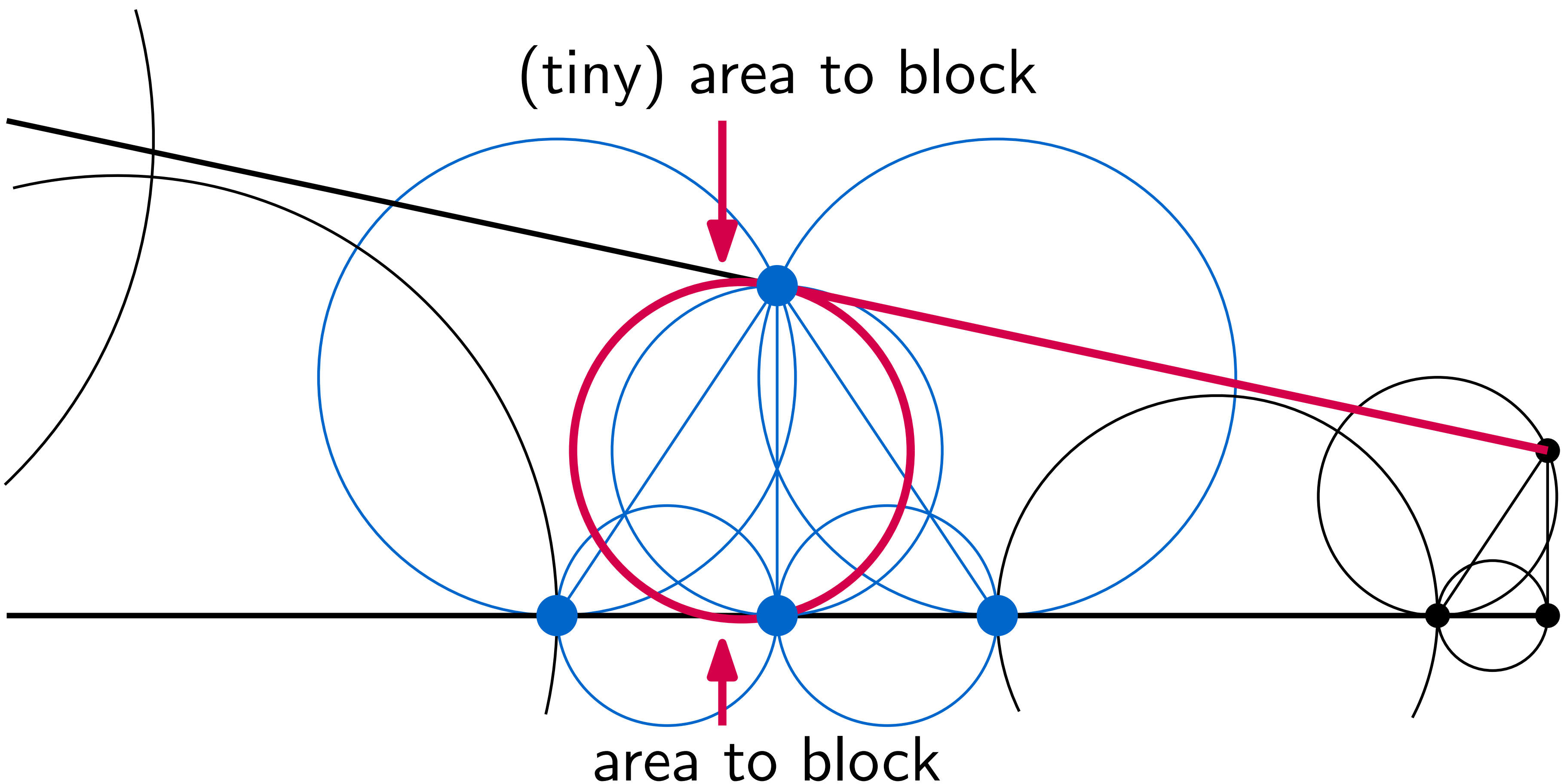

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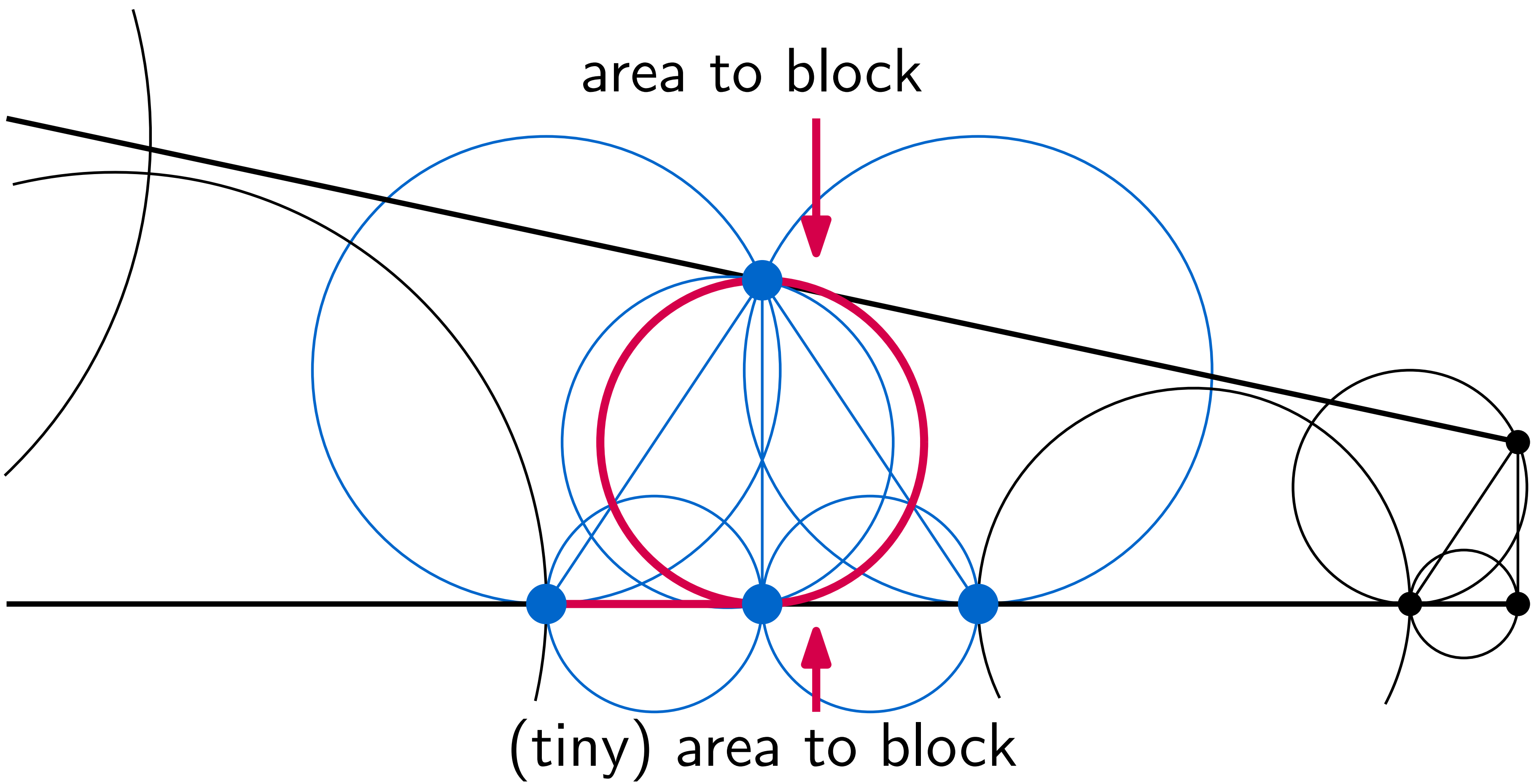

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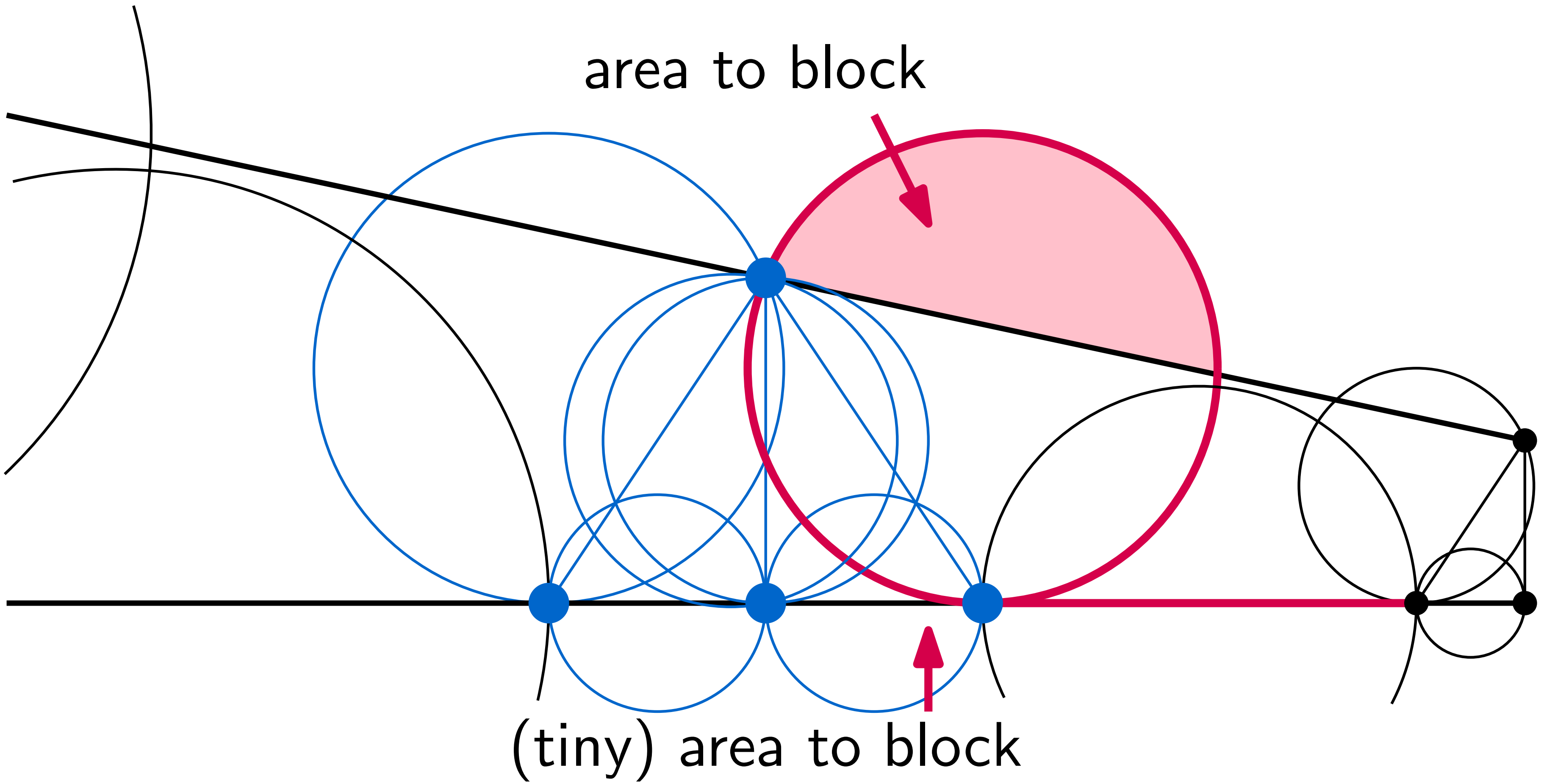

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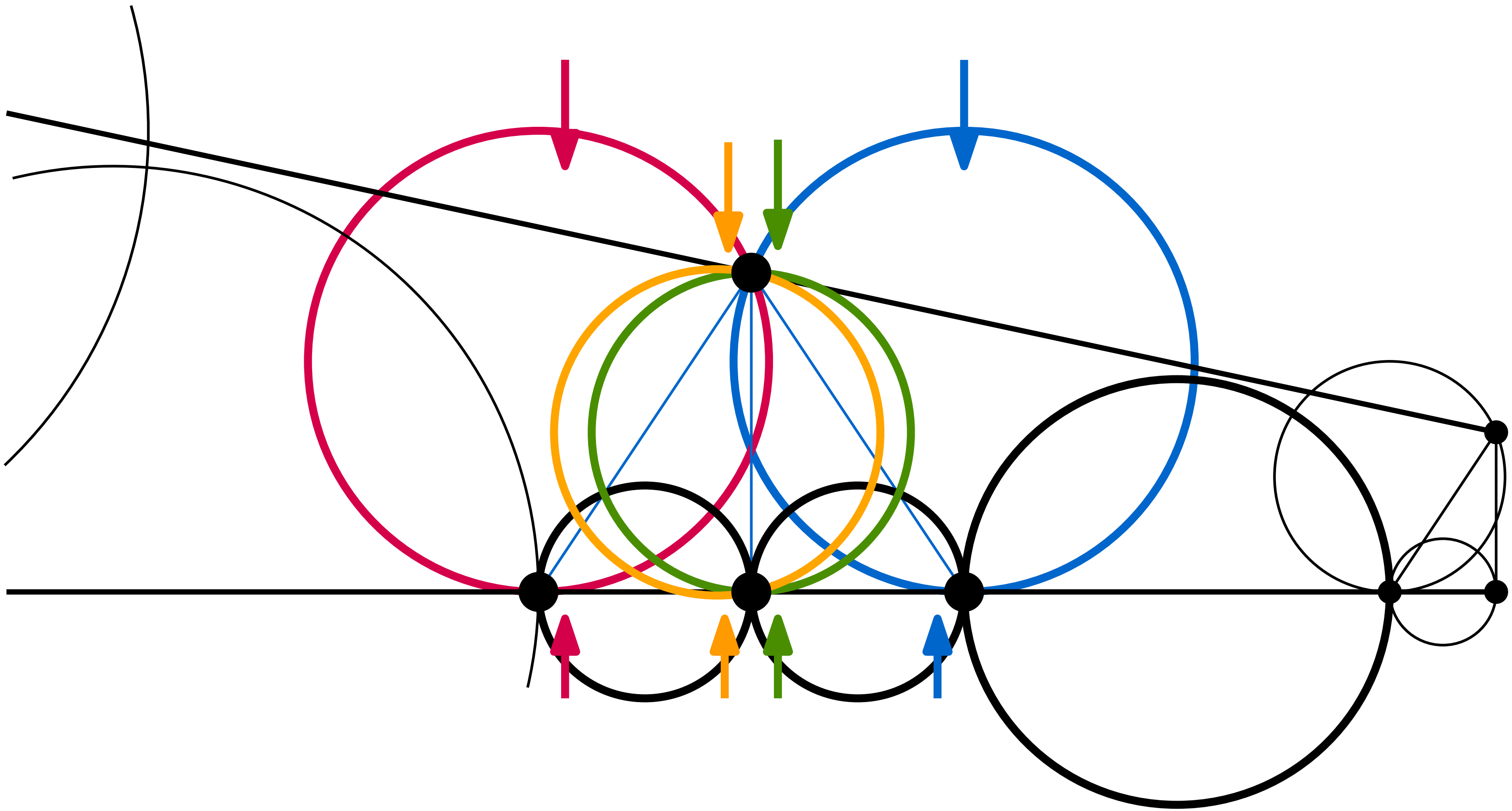

thank you for your attention