Wagner's Method for the Bichromatic Triangle Conjecture

Fernando Cortés Kühnast

02.02.2023

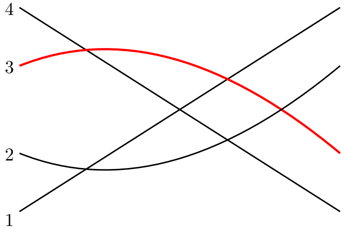
Outline

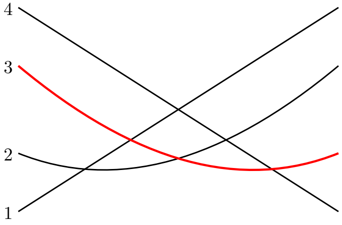
Preliminaries

Arrangements of Pseudolines

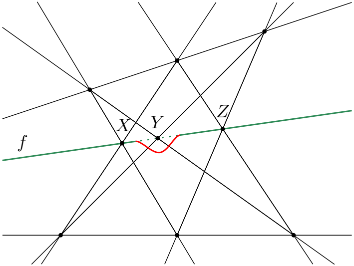
Definition

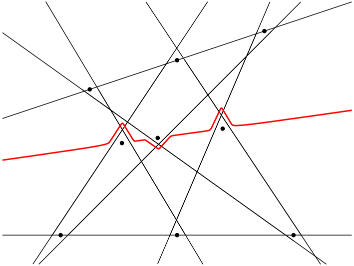
An *arrangement of pseudolines* in $\mathbb{R}^2$ is a finite family of simple x-monotone curves, each of which approaches infinity in both directions and any two of which intersect in exactly one point, where they cross. It is called simple if no three lines intersect in a point.



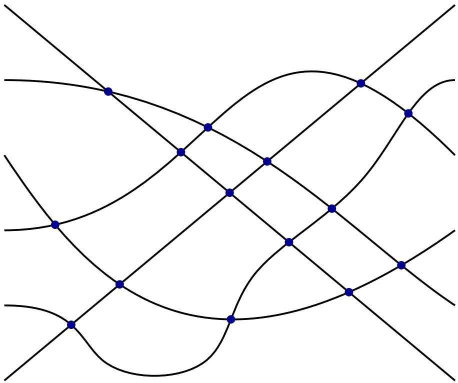


Example: Non-Pappus Arrangement





The Bichromatic Triangle Conjecture



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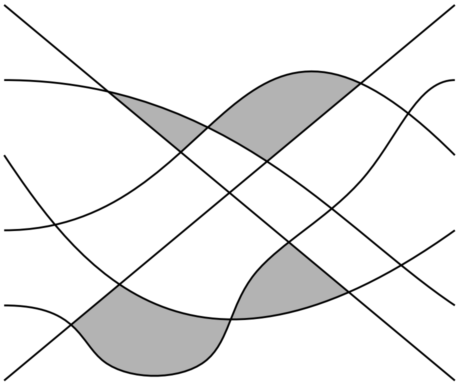
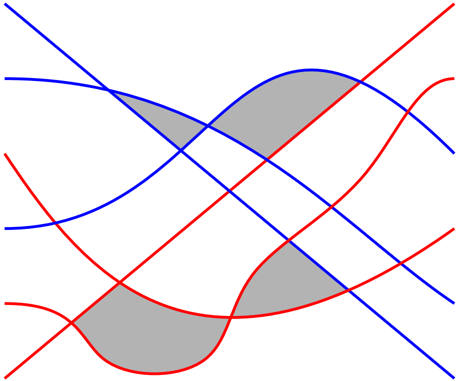


The Bichromatic Triangle Conjecture

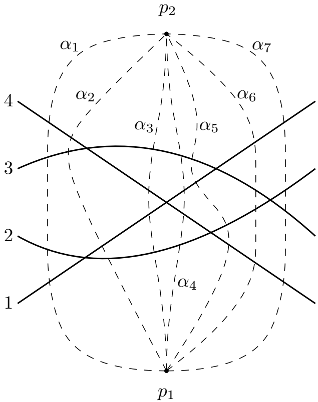
Conjecture

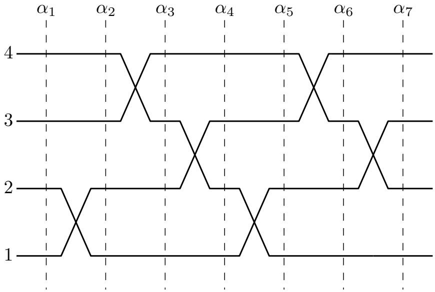
If one colors an arrangement of at least 3 pseudolines with two colors, such that both colors are used at least once, then there must be a bichromatic triangle cell.



Applying Wagner's Method

Encoding Arrangements of Pseudolines





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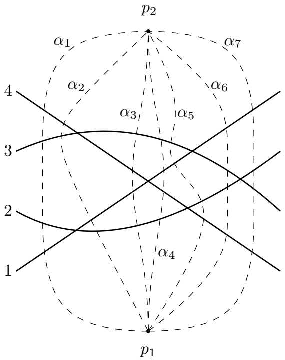

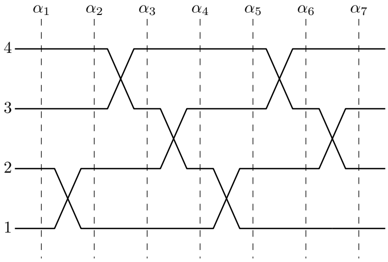

- allowable sequence: $\begin{pmatrix}4\\3\\2\\1\end{pmatrix} \begin{pmatrix}4\\3\\1\\2\end{pmatrix} \begin{pmatrix}3\\4\\1\\2\end{pmatrix} \begin{pmatrix}3\\1\\4\\2\end{pmatrix} \begin{pmatrix}3\\1\\2\\4\end{pmatrix} \begin{pmatrix}1\\3\\2\\4\end{pmatrix} \begin{pmatrix}1\\2\\3\\4\end{pmatrix}$
- reflection network: $[1,2][3,4][2,3][1,2][3,4][2,3]$

Checking the Conjecture

Definition

For any arrangement $\mathcal{A}$ let $G_\ell(A)$ be the graph with pseudolines from $\mathcal{A}$ as vertices with an edge between two if they are incident to the same triangle cell.

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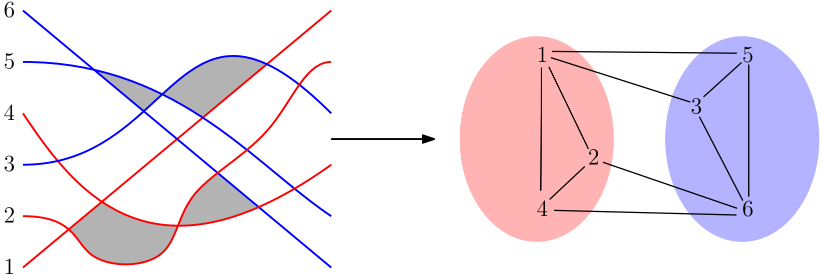

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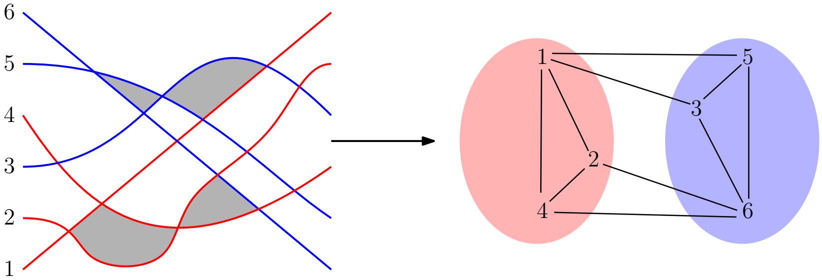
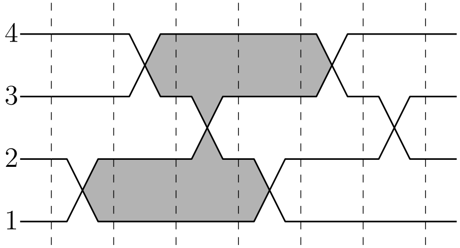


Observation

An arrangement $\mathcal{A}$ admits a coloring without bichromatic triangles if and only if $G_\ell(\mathcal{A})$ is disconnected.

Getting $G_\ell(\mathcal{A})$ in Practice



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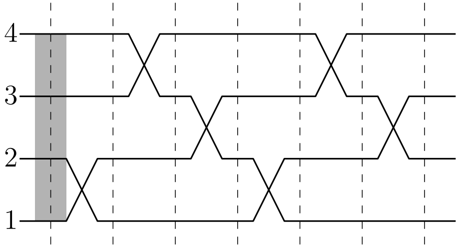

Triangles: $\{\}$

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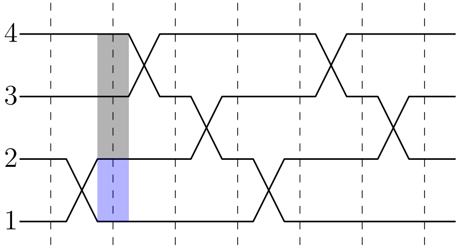

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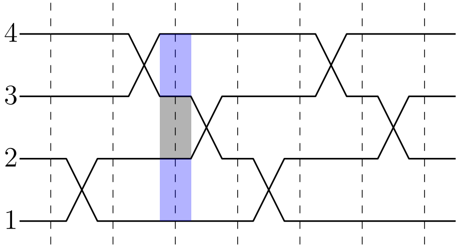

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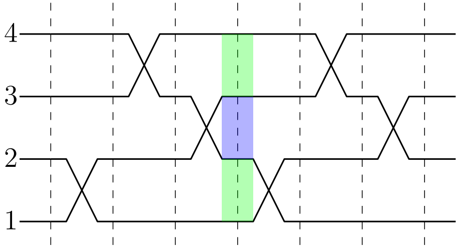

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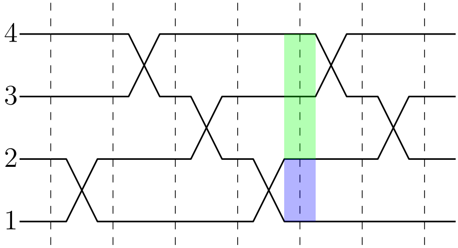
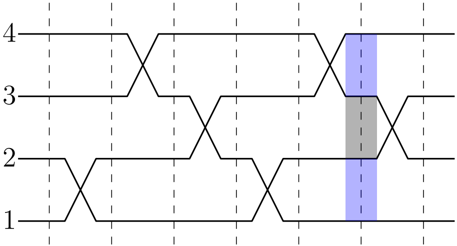


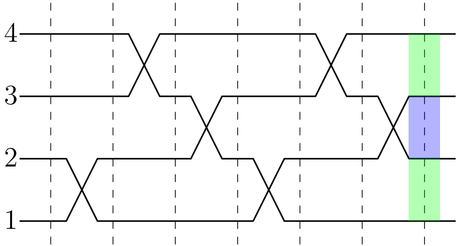
Triangles: $\{(1, 2, 4)\}$

Getting $G_\ell(\mathcal{A})$ in Practice



Triangles: $\{(1,2,4),(1,3,4)\}$

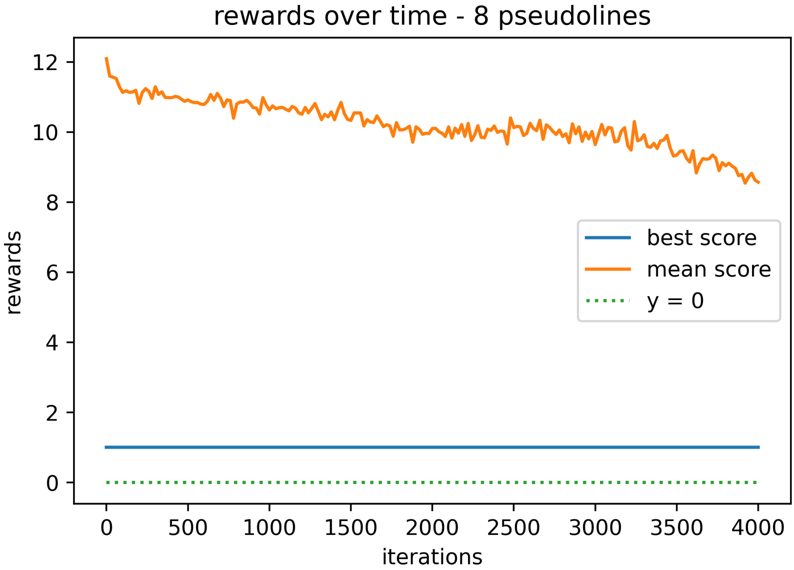
Getting $G_\ell(\mathcal{A})$ in Practice



Triangles: $\{(1, 2, 4), (1, 3, 4)\}$

Results

Results 8 Pseudolines



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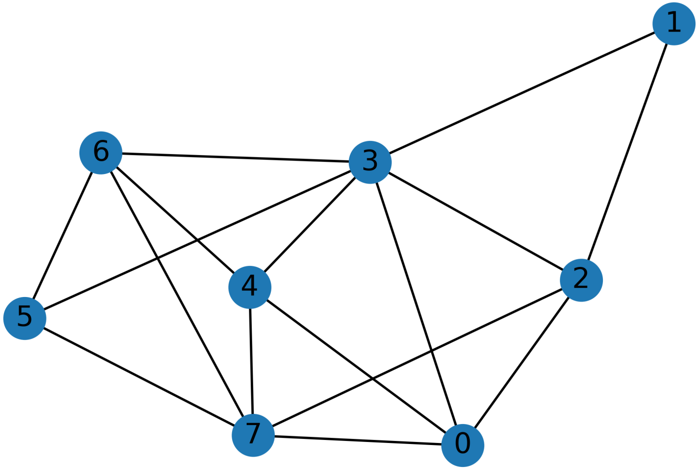
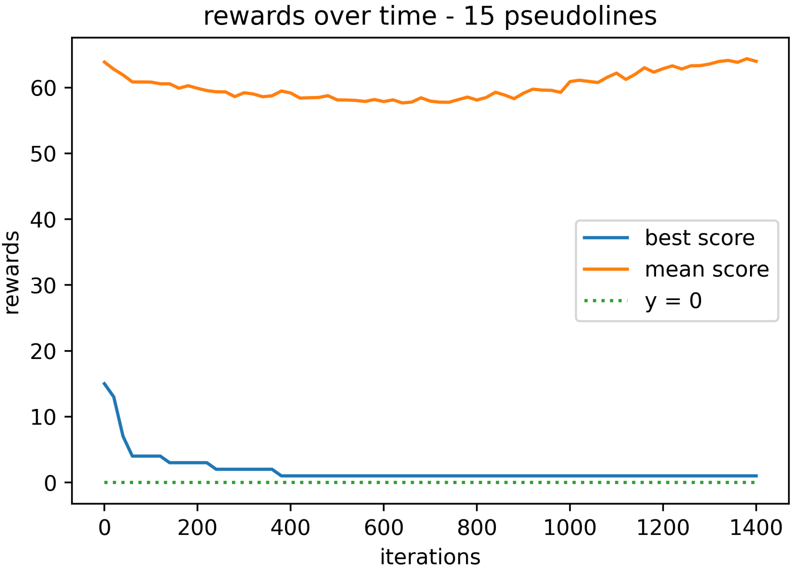
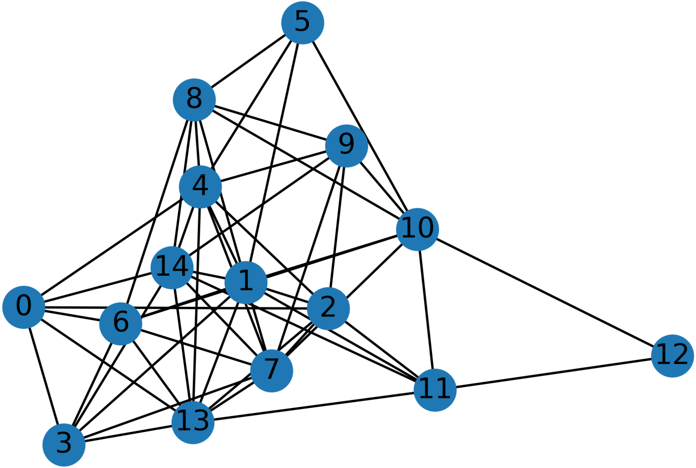


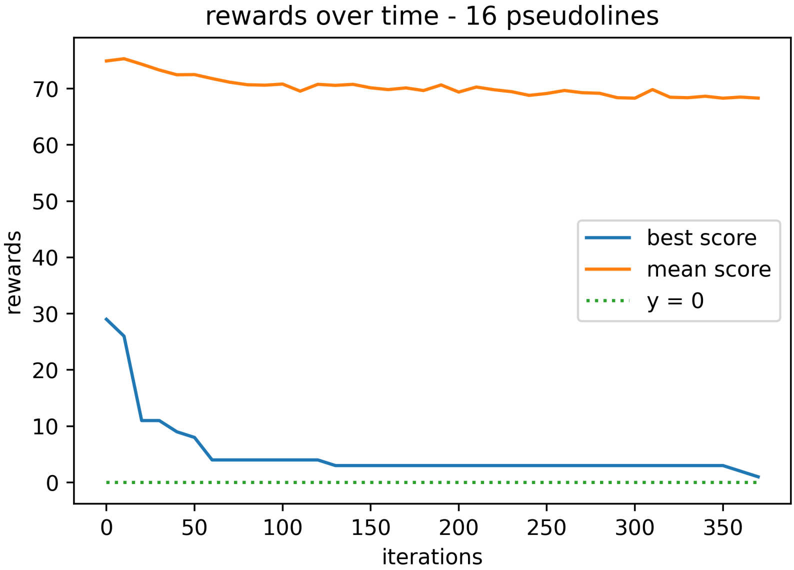
Results 15 Pseudolines



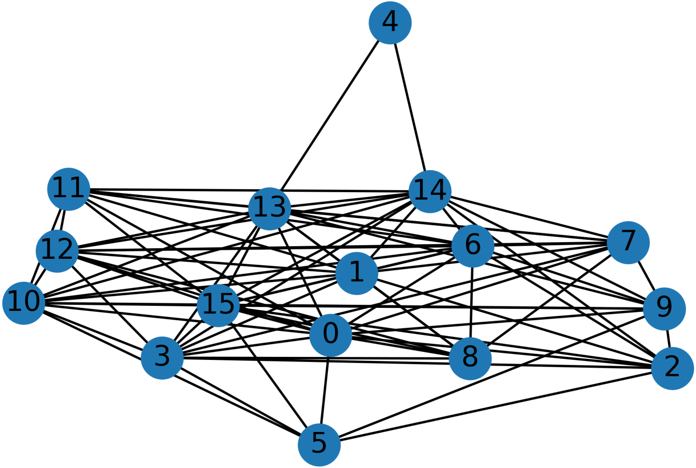
Results 15 Pseudolines



Results 16 Pseudolines



Results 16 Pseudolines



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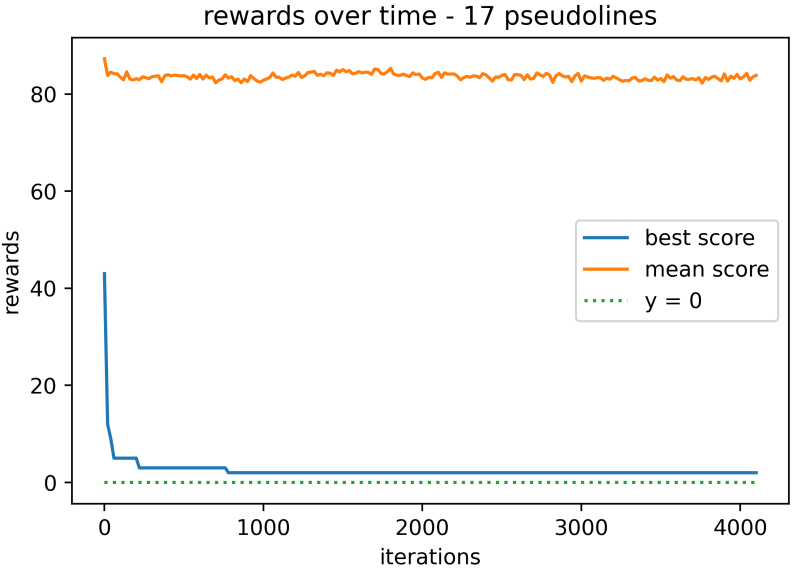

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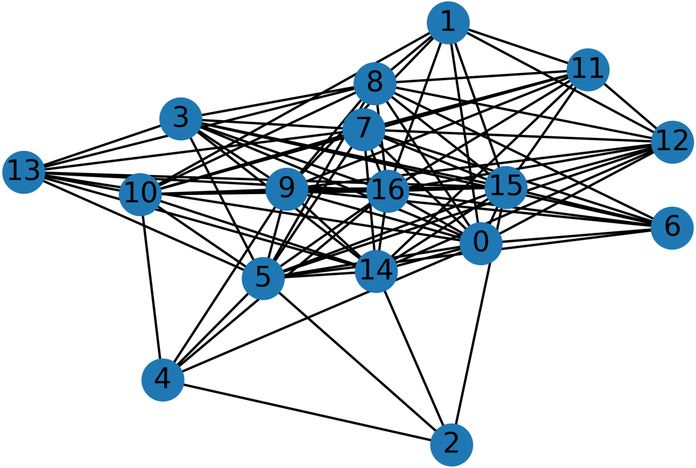
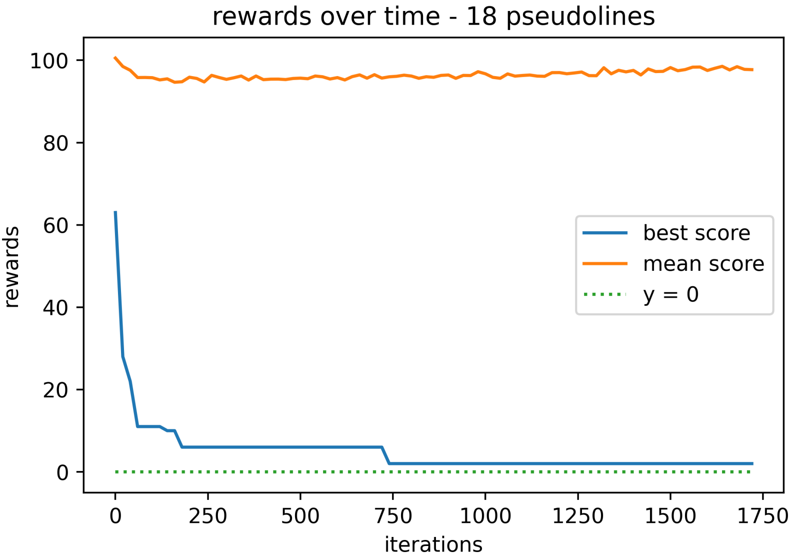


Results 18 Pseudolines



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