Panizzut - Roch
Technical University Berlin
Summer semester 2022

Discrete Geometry I

Exercise Sheet 9

Exercise 1 - Vertices of LP

Let $P(A,b) \subseteq \mathbb{R}^n$ be a feasible region of a linear program. Show that for $v \in \partial P$ the following statements are equivalent:

a) The point v is a vertex of P.

b) The matrix $A'(v)$ of active conditions has full rank n.

Exercise 2 - Farkas' Lemma

Let A be an $m \times n$-matrix and $b \in \mathbb{R}^m$. Then *either* the system of inequalities

$$Ax = b,\ x \geq 0\ (x \in \mathbb{R}^n)$$

or the system of inequalities

$$A^T z \geq 0,\ b^T z < 0\ (z \in \mathbb{R}^m)$$

has a solution.

Hint: If the first system has no solution, then the set $\{y \in \mathbb{R}^m : Ax = y \text{ for an } x \geq 0\}$ can be strictly separated from the vector b.

Exercise 3 - Dual LPs

Construct different dual pairs of linear programming problems

$$\max\{cx : Ax \leq b\} \text{ and } \min\{yb : yA = c,\ y \geq 0\}$$

with the following additional characteristics:

a) the primal problem is unbounded and the dual problem is feasible;

b) the primal problem is infeasible and the dual is unbounded;

c) both problem are infeasible.