



Discrete Geometry I

Exercise Sheet 8

Exercise 1 - Graphs of polytopes

The diameter of a graph G is the smallest number δ such that any two vertices in G can be connected by a path with at most δ edges. For $m > n \geq 2$, let $\Delta(n, m)$ be the maximal diameter of the graph of a n -dimensional polytope with at most m facets (see Exercise 4 Sheet 6). Determine $\Delta(2, m)$.

Exercise 2 - Cyclic polytopes

Show that each edge of the cyclic polytope $Z_4(m)$, $m \geq 6$, is incident with either three, four or $m - 2$ facets, and that edges of all these types occur.

Exercise 3 - Incidence matrices

- a) Show that the face lattice of a polytope is completely described by the facets.
- b) Show that two polytopes are combinatorially equivalent if and only if there exists an ordering of their vertices and facets such that their corresponding incidence matrices are equal.