Panizzut - Roch
Technical University Berlin
Summer semester 2022

Discrete Geometry I

Exercise Sheet 7

Exercise 1 - Cubical polytopes

A polytope is called *cubical* if all its proper faces are combinatorially equivalent to cubes. Show that the following inequality holds for the f-vector of a cubical polytope P

$$f_1(P) + 2f_2(P) + 2^2 f_3(P) + \cdots 2^{n-2} f_{n-1}(P) \leq \binom{f_0(P)}{2}.$$

Exercise 2 - Zonotopes

The Minkowski sum (Exercise 1 Sheet 2) $[p_1, q_1] + \cdots + [p_k, q_k]$ of a finite number of line segments with $p_i, q_i \in \mathbb{R}^n$ is a *zonotope*. Show that the zonotopes generated by k segments are exactly the images of the standard cube $[-1, 1]^k$ under affine maps.

Exercise 3 - Simple polytopes

Let P be a simple n-polytope.

a) Check that the Dehn-Sommerville equations for $n = 4$ are equivalent to $f_1(P) = 2f_0(P)$ and the Euler formula.

b) For $n = 5$ find a linear relation that follows from the Dehn-Sommerville equations but it is independent from the Euler formula and $2f_1(P) = 5f_0(P)$.