Panizzut - Roch
Technical University Berlin
Summer semester 2022

Discrete Geometry I

Exercise Sheet 4

Exercise 1 - Simplicial polytopes

Show that for any d-dimensional polytope P, the following conditions are equivalent:

a) Every facet of P is a simplex, i.e., P is *simplicial*.

b) Every proper face of P is a simplex.

c) Every facet of P has exactly d vertices.

d) Every k-face has exactly $k+1$ vertices, for $k \leq d-1$.

Exercise 2 - Simplicial and simple polytopes

A d-dimensional polytope is *simple* if every vertex is adjecent to d vertices.
Show that if a polytope is both simple and simplicial then it is either a simplex or a n-gon.

Exercise 3 - Combinatorially equivalent polytopes

Give an example of two combinatorially equivalent polytopes such that they are not affinely isomorphic.

Exercise 4 - Face lattice of a polytope

The face lattice $(\mathcal{F}(P), \subseteq)$ will be defined in the lecture on Monday, May 16.
Show that $(\mathcal{F}(P), \subseteq)$ satisfies the following conditions:

a) There exists a uniquely determined smallest and largest face of P.

b) For two arbitrary faces $F, G \in \mathcal{F}(P)$ there exists a uniquely determined smallest face $F \vee G$ such that $F \subseteq F \vee G$ and $G \subseteq F \vee G$.

c) For two arbitrary faces $F, G \in \mathcal{F}(P)$ there exists a uniquely determined largest face $F \wedge G$ such that $F \supseteq F \wedge G$ and $G \supseteq F \wedge G$.