



Discrete Geometry I

Exercise Sheet 3

Exercise 1 - Faces of a polytope

For a polytope P show that:

- The intersection of a set of faces of P is a face of P .
- Every ridge of P is the intersection of exactly two facets of P .
A ridge is face of dimension $\dim P - 2$.
- If G is a face of P and F is a face of G , then F is a face of P .

Exercise 2 - 3-simplices

Given a 3-dimensional polytope such that every two vertices are adjacent, i.e., connected by an edge, show that it is a simplex.

Exercise 3 - Helly's theorem

Let C_1, C_2, \dots, C_n be convex sets in \mathbb{R}^d , $n \geq d+1$. Suppose that the intersection of every $d+1$ sets is nonempty. Show that the intersection of all the C_i is not empty.

Exercise 4 - Faces of hypercube

Let $C_d \subset \mathbb{R}^d$ be the d -dimensional cube. Enumerate its $3^d + 1$ proper and non-proper faces. Show that the non-empty faces are naturally associated with the sign vectors in $\{+, -, 0\}^d$.

Exercise 5 - H -polytope

Determine the vertices of the following polytope P ,

$$P = \{(x, y, z) \in \mathbb{R}^3 \mid z \geq 0, x \geq 0, -x + y \geq 0, -x - y \geq -2, -3x - y - 2z \geq -4\}$$