§20 More on Complexity of Project Scheduling with Resource Constraints

Have seen complexity results for machine scheduling problems

$\underbrace{\text{Project Scheduling}}_{\text{partial order } G}$ with $\underbrace{\text{Resource Constraints}}_{\text{system of forbidden sets}}$ is "much harder"

$\hat{=}$ jobs may require several limited resources

Will show hardness of approximation and reoptimization already for C_{max} and deterministic processing times

Problem is called RCPSP $\hat{=}$ resource constrained project scheduling problem"

20.1 Theorem (Folklore)

RCPSP is as hard to approximate as vertex coloring, i.e.

Unless P = NP, there is no approximation algorithm with a performance guarantee of $n^{1-\varepsilon}$ for every $\varepsilon \in [0,1[$, n = # of jobs

Proof: Transform instance of COLORING to instance of RCPSP

COLORING:	$\Longrightarrow$	instance I' of RCPSP
Given: undirected graph G, number $k \in \mathbb{N}$		Given: $V = V(G)$ with $x_j = 1 \ \forall j \in V$; no precedence constraints; $\mathcal{F} = E(G)$, every edge is forbidden
Question: $\exists$? vertex coloring with $\leq k$ colors		Question: $\exists$? schedule with $C_{max} \leq k$

Then vertices of the same color of G correspond to jobs in the same time slot

$\Rightarrow$ yes instances of COLORING $\hat{=}$ yes instances of I'

$\Rightarrow$ the inapproximability result for graph coloring [Zuckermann 2007] applies $\square$

For the complexity of reoptimization, we consider two UPDATE problems

UPDATE AFTER ADDING A FORBIDDEN SET (UPDATE - ADD)

Given: an optimal schedule for an instance I of RCPSP

a new forbidden set F

Wanted: an optimal schedule for the RCPSP after adding the forbidden set F

20.2 Theorem (Schäffter 1997)

UPDATE - ADD is NP-hard in the weak sense

Proof: Reduction from PARTITION

Given: numbers $a_1, \dots, a_n \in \mathbb{N}$ with $\sum_i a_i = 2b$, $b \in \mathbb{N}$

Question: $\exists^?\ I \subseteq \{1, \dots, n\}$ with $\sum_{i \in I} a_i = b$

From PARTITION, construct the following instance I of RCPSP

jobs: $V = \{1, \dots, n\} \cup \{d_1, d_2, d_3\}$

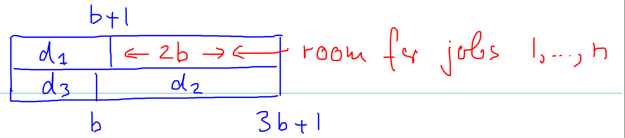
processing times: $a_1 \dots a_n$ $b+1$, $2b+1$, b

precedence constraints: none

forbidden sets: $\hat{=}$ 2-machine problem + all $\{d_3, j\}$, $j \in \{1, \dots, n\}$

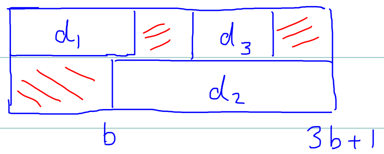
new forbidden set: $F = \{d_1, d_3\}$ gives instance I'

The given optimal schedule for I is :



Consider now I'.

$\{d_1, d_3\}$ forbidden $\Rightarrow$ d_3 must be parallel to d_2 if $C_{max} = 3b+1$



red space is b on both machines

So deciding if $C_{max} \le 3b+1$ is as hard as solving PARTITION $\square$

UPDATE AFTER DELETING A FORBIDDEN SET (UPDATE-DELETE)

Given: an optimal schedule for an instance I of RCPSP

a forbidden set F of I

Wanted: an optimal schedule of the RCPSP after deleting F

20.3 Theorem (Schäffler 1997)

UPDATE-DELETE is NP-hard in the strong sense

Proof: Reduction from VERTEX COLORING OF PLANAR GRAPHS WITH MAXIMUM DEGREE 4

Given: planar undirected graph G with max degree $\Delta \le 4$

Question: $\exists$? a vertex coloring of G with 3 colors

This problem is strongly NP-complete [Garey, Johnson, Stockmeyer 1979]

This is interesting in the view of

THEOREM OF Brooks (1941)

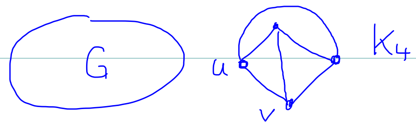
Every graph $\neq K_n$ and C_{2k+1} can be colored with Δ colors

complete graph with n vertices — odd cycle with 2k+1 vertices — max degree

So if the given planar graph G with $\Delta \le 4$ is not bipartite (2 colorable) then the answer can only be 3 colors or 4 colors, but this is NP-hard to decide (Note that this follows also from the 4-color theorem)

Reduction to RCPSP:

Add to G (planar, $\Delta \le 4$) a K_4 as separate connected component



$\hat{=}\ \bar{G}$

Transform $\bar{G}$ to instance I of RCPSP as in the proof of Theorem 20.1

Given: $V = V(\bar{G})$, $x_j = 1 \ \forall j$

no prec

$\mathcal{F} = E(\bar{G})$

Since $\bar{G}$ contains a K_4, it needs 4 colors

Since $\Delta(\bar{G}) \le 4$, 4 colors are optimal

So I has an optimal schedule with 4 colors and we assume that we know one.

Now consider instance I' when the forbidden set $\{u,v\}$ is deleted.

Now it is no longer clear if 4 colors are needed

So $\underbrace{\text{coloring } \bar{G} - uv \text{ optimally}}$ is as hard as coloring G optimally

$\Updownarrow$

constructing an optimal schedule for I' $\square$