From random walk trajectories to random interlacements

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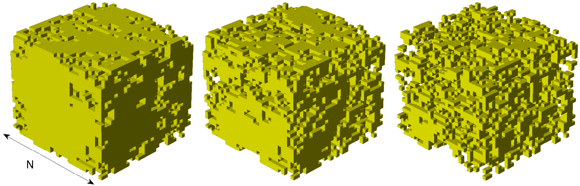
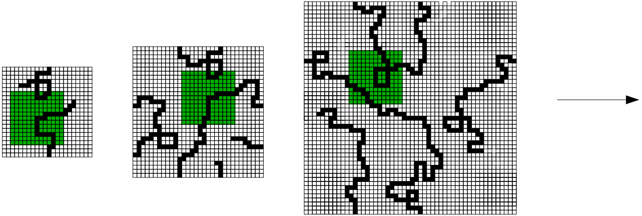
Motivation - Porous medium



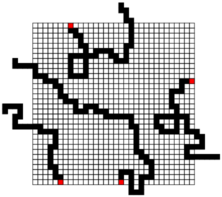
- Random walk on $(\mathbb{Z}/N\mathbb{Z})^d$
- Large N, $d \geqslant 3$,
- In time $\sim N^d \log N \longrightarrow$ all covered
- In time $\sim N^d \longrightarrow$ proportion covered
- Abrupt connectivity transitions (simulations)
- Geometry of sites left: unknown

1. Fix a box in each torus (constant size)
2. Fix $u \geq 0$. Run the random walk up to time uN^d



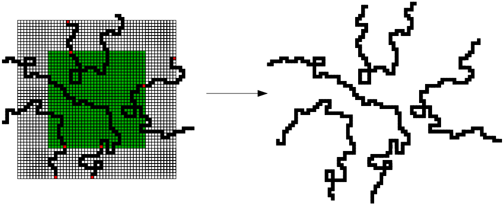
Theorem - Windisch (2008)

Law of visited sites in box converges



1. Poisson start points in boundary
2. Independent walks from them

Extended to $\mathbb{Z}^d$



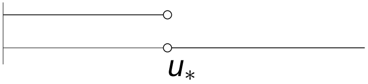
Random Interlacement - Sznitman (2007)

Poisson soup of doubly infinite trajectories on $\mathbb{Z}^d$ modulo time shift

1. Law Q^u on $\{0,1\}^{\mathbb{Z}^d}$
2. u gives intensity
3. Ergodic under translations
4. Disconnection of discrete cylinder

Geometry of vacant sites

Ergodicity $\Longrightarrow$ $Q^u[\exists \text{ infinite component}] = 0 \text{ or } 1$



Theorem - Sznitman (2007), Sidoravicius-Sznitman (2008)

In $\mathbb{Z}^d$, $(d \geqslant 3)$ $\quad 0 < u_* < \infty$

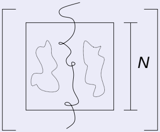
Theorem - T. (2009)

In $\mathbb{Z}^d$, $(d \geqslant 3)$ $\quad$ Vacant infinite component is unique.

Separation events

Theorem - T. (2009)

($d \geqslant 5$) For u small, there is a δ such that

Q^u 

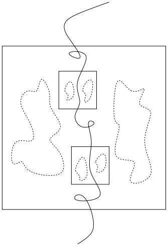
$\leqslant c \exp\{-c' N^{\delta}\}.$

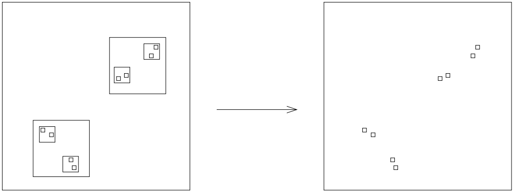
Consequences

- The infinite cluster is ‘dense’
- Typical components are either: small or infinite
- Could give some results in the torus

Strategies

1 - Renormalization



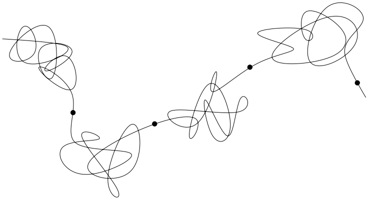


$$q_k \leq 1000^{2^k}(q_1^{2^k} + c_k)$$

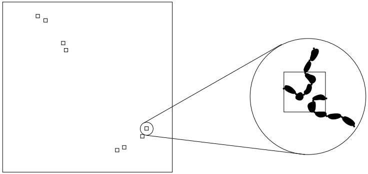
2- Walking around sausages

We only need to bound q_1 (probability of separation in a small box)

If $d \geq 5$, a random walk has several *cut-times*



And the random walk can be regarded as a 'sausage'



- There will be less than 100 random walks in each of these ‘grains’
- With high probability they are far apart from each othe ($d \geq 5$)
- There is not separation in this case:

any two big sets can be linked by a path
that ‘travels through the skins of the sausages’

Don’t build dikes with sausages!

V. Sidoravicius, A.S. Sznitman
Percolation for the vacant set of random interlacements
Comm. Pure Appl. Math., 62(6), 831-858 (2009)

A.S. Sznitman
Vacant set of random interlacements and percolation
to appear in the *Annals of Mathematics* (2007)

A. Teixeira
On the uniqueness of the infinite cluster of the vacant set of random interlacements
Annals of Applied Probability, 19, 1, 454-466 (2009)

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Interlacement percolation on transient weighted graphs
Electronic Journal of Probability, 14, 1604-1627 (2009)

A.Teixeira
On the size of a finite vacant cluster of random interlacements with small intensity
submitted (2009)

Thanks!