Using Duality in the Partial Duplication Random Graph

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with Peter Pfaffelhuber

Albert-Ludwigs-University, Freiburg

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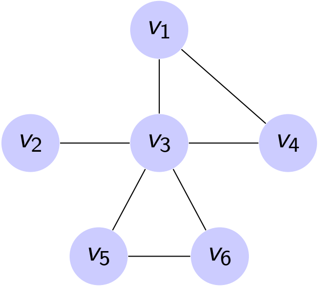
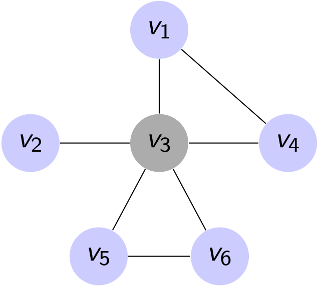
Partial duplication

Initial graph



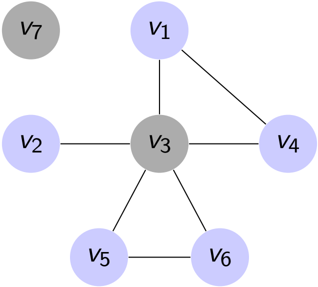
Partial duplication

Choice of a vertex



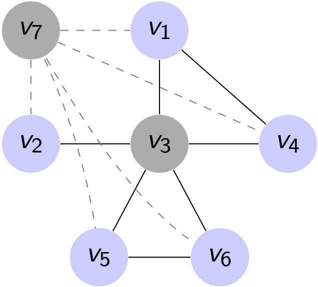
Partial duplication

Addition of a new vertex



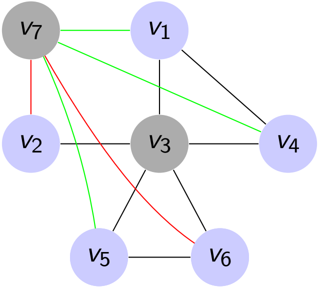
Partial duplication

Duplication of the connections

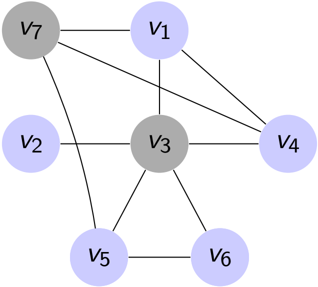


Partial duplication

Deletion of new edges with probability $1 - p$ respectively

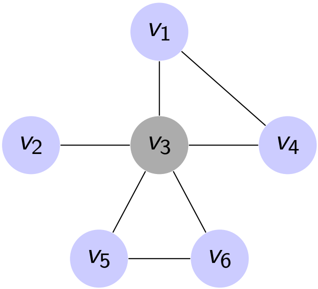


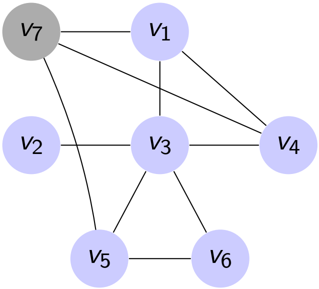
Partial duplication



Summary

Time step





Notation

- $p > 0$, G_{n_0} connected
- n_0: size of initial graph, ≥ 2
- G_n: partial duplication graph at time $n \geq n_0$
- $\mathcal{F}_n := \sigma(G_{n_0}, \ldots, G_n)$
- $F_k(n) := |\{v \in G_n \mid \deg_{G_n}(v) = k\}|$
- $F_k^\circ(n) := F_k(n)/n$

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Simple deductions

only one connected component

many singletons

- If there is a singleton at time n,
- it will be copied with probability $1/n$.
- Borel-Cantelli: almost surely infinitely often
- $\mathbb{E}[F_0^\circ(n+1)] \geq \mathbb{E}[F_0^\circ(n)]$
- $\mathbb{E}[F_0^\circ(n)] \overset{n\to\infty}{\longrightarrow} f_0^\circ \leq 1$

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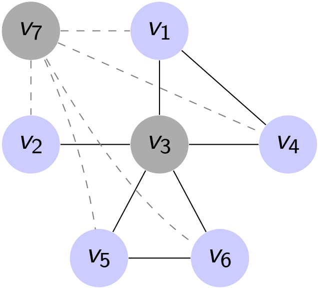
Emergence of singletons



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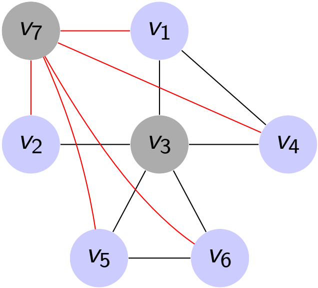

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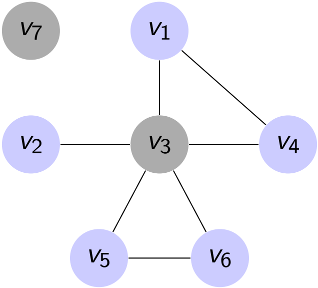

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What is f_0°?

Gulkan Bebek et al, 2006

The degree distribution of the generalized duplication model

- $f_0^\circ = 1$ if $p \leq \frac{1}{2}$

what about $p > \frac{1}{2}$?

- $\liminf_n \mathbb{E}[F_k^\circ(n)] = 0$ für alle $k > 0$.
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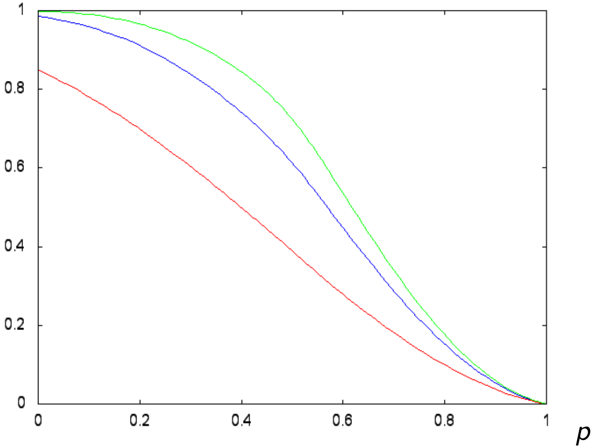

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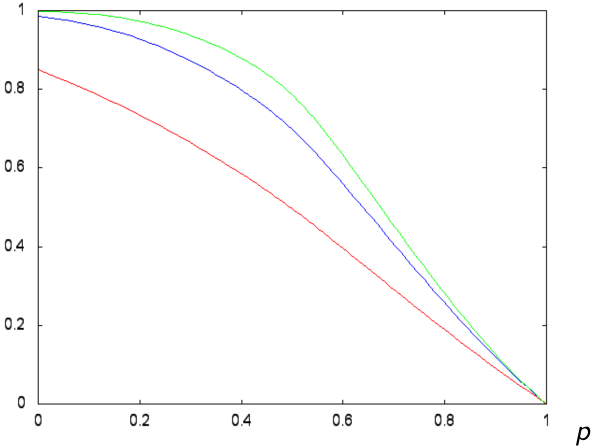

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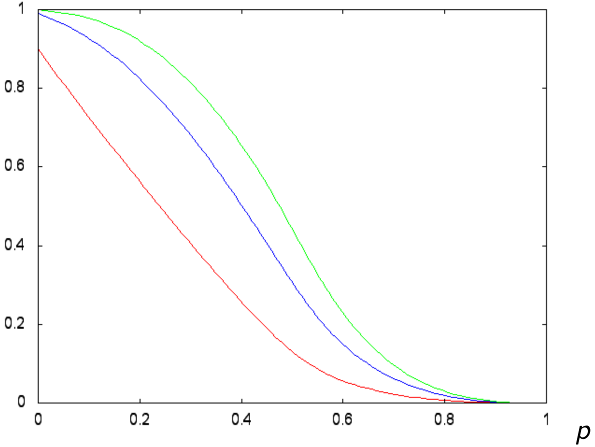

Theorem

let $H_x(n) = \sum_{k \geq 0} (1-x)^k F_k^\circ(n)$, probability generating function

- $\mathbb{E}[H_x(n)] \overset{n\to\infty}{\longrightarrow} h_\infty$ for each $x > 0$
- $h_\infty = 1$ if $p \leq p^*$ and $h_\infty < 1$ if $p > p^*$
- $p^* \approx 0.56714$ solves $pe^p = 1$

$$\Rightarrow \; f_0^\circ = \lim_{n\to\infty} \mathbb{E}[F_0^\circ(n)] = \lim_{n\to\infty} \mathbb{E}[H_1(n)] = h_\infty$$

and for $k > 0$:

$$\mathbb{E}[F_k^\circ(n)] \cdot (-1)^k k! = \frac{\partial^k}{\partial x^k} \mathbb{E}[H_x(n)]\Big|_{x=1} \overset{n\to\infty}{\longrightarrow} \frac{\partial^k}{\partial x^k} h_\infty = 0$$

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Recursion for $H_x(n) := \sum_k F_k^\circ(n)(1-x)^k$

- $\mathbb{E}[F_k(n+1) - F_k(n)|\mathcal{F}_n]$

  $$= -pkF_k^\circ(n) + p(k-1)F_{k-1}^\circ(n) + \sum_{\ell \geq k} F_\ell^\circ(n)\binom{\ell}{k}p^k(1-p)^{\ell-k}$$

- multiplying by $(1-x)^k$ and summing over k:

  $$\mathbb{E}[(n+1)H_x(n+1) - nH_x(n)|\mathcal{F}_n]$$

  $$= px(1-x)\frac{\partial}{\partial s}H_s(n)\Big|_{s=x} + H_{px}(n)$$

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Continuous time

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let $H_x(t) = H_x(N_t)$

- $(N_t)_{t\geq 0}$ counting process starting in n_0
- jumping with rate $N_t + 1$ at time t

$\mathbb{E}[H_x(t+dt) - H_x(t)|\mathcal{F}_t]$

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Continuous time

$$\mathbb{E}[H_x(n+1) - H_x(n)|\mathcal{F}_n] = \frac{1}{n+1}\Big(- H_x(n) + H_{px}(n) + px(1-x)\frac{\partial}{\partial s}H_s(n)\Big|_{s=x}\Big)$$

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$$\mathcal{G}_{\mathcal{X}} f(x) = f(px) - f(x) + px(1-x)f'(x)$$

$Y_t := -\log X_t$, $\mathcal{Y} := (Y_t)_t$

$\Rightarrow \; X_t \to 0$ iff $Y_t \to \infty$

and $\; G_{\mathcal{Y}} f(y) = f(y + \log \frac{1}{p}) - f(y) - p(1 - e^{-y}) f'(y)$

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$p \leq \log \frac{1}{p}$:

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$p \leq p^*$:

- $X_t \to 0$
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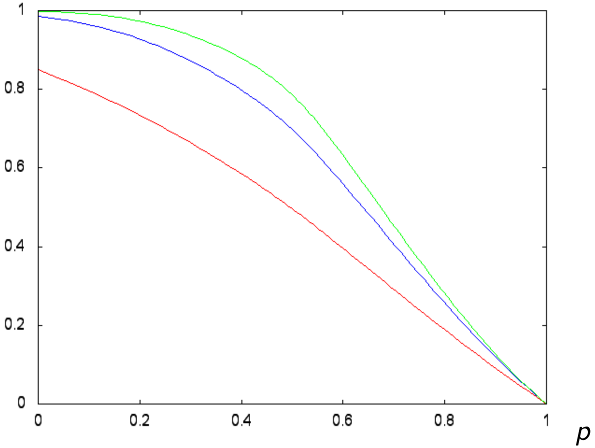
$F_0^\circ(n),\ n \in \{10^2, 10^3, 10^4\}$



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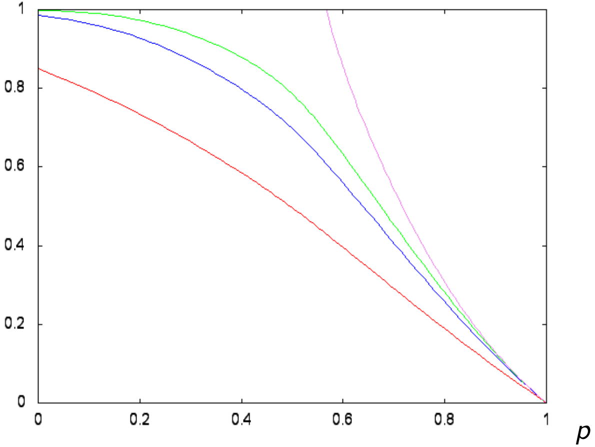

Further possible applications

asymptotic size of connected component for $p \leq p^*$

- $\mathbb{E}[1 - F_0^\circ(t)] \sim c\mathbb{E}[X_t]$

using method for generalized duplication models

or other models (preferential attachment?)

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More open questions

degrees:

- $\deg_{G_n}(v) \sim n^p D_v$ for all v
- $\mathbb{E}[D_v^k] = \frac{\Gamma(d_v+k)}{\Gamma(d_v)} \cdot \frac{\Gamma(n_v)}{\Gamma(n_v+pk)}$
- maximum degree?

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- $p \searrow 0$: preferential attachment

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More open questions

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Improved duplication models for proteome network evolution

- introduced fitting modified model (*sequence similarity*)