

THE CARMICHAEL NUMBERS UP TO 10^{19} (ABSTRACT)

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A Carmichael number N is a composite number N with the property that for every b prime to N we have $b^{N-1} \equiv 1 \pmod{N}$.

For background on Carmichael numbers and details of previous computations see [1]: in that paper we described the computation of the Carmichael numbers up to 10^{15} and presented some statistics. These computations have since been extended to 10^{18} [2] and now to 10^{19} , using similar techniques, and we present further statistics.

The principal search was a depth-first back-tracking search over possible sequences of primes factors p_1, \dots, p_d satisfying p_i coprime to $p_j(p_j - 1)$ for $i \neq j$.

We also employed the variant based on proposition 2 of [1] which determines the finitely many possible pairs (p_{d-1}, p_d) from P_{d-2} . In practice this was useful only when $d = 3$.

Finally we employed a different search over large values of p_d , in the range $2 \cdot 10^6 < p_d < 10^{9.5}$, using the property that $P_{d-1} \equiv 1 \pmod{p_d - 1}$.

We have shown that there are 3381806 Carmichael numbers up to 10^{19} , all with at most 12 prime factors. We let $C(X)$ denote the number of Carmichael numbers less than X and $C(d, X)$ denote the number with exactly d prime factors. Table 1 gives the the values of $C(d, X)$ for X in powers of 10 up to 10^{19} .

X	3	4	5	6	7	8	9	10	11	12	total
3	1	0	0	0	0	0	0	0	0	0	1
4	7	0	0	0	0	0	0	0	0	0	7
5	12	4	0	0	0	0	0	0	0	0	16
6	23	19	1	0	0	0	0	0	0	0	43
7	47	55	3	0	0	0	0	0	0	0	105
8	84	144	27	0	0	0	0	0	0	0	255
9	172	314	146	14	0	0	0	0	0	0	646
10	335	619	492	99	2	0	0	0	0	0	1547
11	590	1179	1336	459	41	0	0	0	0	0	3605
12	1000	2102	3156	1714	262	7	0	0	0	0	8241
13	1858	3639	7082	5270	1340	89	1	0	0	0	19279
14	3284	6042	14938	14401	5359	655	27	0	0	0	44706
15	6083	9938	29282	36907	19210	3622	170	0	0	0	105212
16	10816	16202	55012	86696	60150	16348	1436	23	0	0	246683
17	19539	25758	100707	194306	172234	63635	8835	340	1	0	585355
18	35586	40685	178063	414660	460553	223997	44993	3058	49	0	1401644
19	65309	63343	306310	849564	1159167	720406	196391	20738	576	2	3381806

TABLE 1. Values of $C(X)$ and $C(d, X)$ for $d \leq 10$ and X in powers of 10 up to 10^{19} .

REFERENCES

1. Richard G.E. Pinch, *The Carmichael numbers up to 10^{15}* , Math. Comp. **61** (1993), 381–391, Lehmer memorial issue.
2. ———, *The Carmichael numbers up to 10^{18}* , April 2006, Unpublished.

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Date: 1 May 2006.