Technical University Berlin **WS 2025**
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8. Exercise Discrete Geometrie II

Deadline: 11.12.2025 (before the Exercise class)

Each answer should be sufficiently proven.

1. Exercise (Puiseux series vs. Puiseux fractions)

Give and example of a (standard) Puiseux series which is not a Puiseux fraction.

2. Exercise (Tropicalization)

Solve the Equation

$$x^5 + tx^4 + t^3x^3 + t^6x^2 + t^{10}x = t^{15}$$

Also solve

$$x^5 + 2x^4 + 8x^3 + 64x^2 + 1024x = 32768$$

3. Exercise (Inclusion)

Let f and g be polynomials in $\mathbb{C}\{\{t\}\}[x_1^{\pm}, \dots, x_n^{\pm}]$ such that $g \in \langle f \rangle$. Show that that their tropical hypersurfaces satisfy $\mathrm{Trop}(g) \subseteq \mathrm{Trop}(f)$.

4. Exercise (Puiseux series)

What is the largest multiplicity of any edge in the tropicalization of any plane curve of degree d. How about surfaces in 3-space?

5. Exercise (Example tropicalization)

Draw trop(V(f)) for the following polynomials $f \in \mathbb{C}\{\{t\}\}[x^{\pm}, y^{\pm}]$:

1. $f = t^3x + (t + 3t^2 + 5t^4)y + t^{-2}$,
2. $f = (t^{-1} + 1)x + (t^2 - 3t^3)y + 5t^4$.
3. $f = t^6x^3 + x^2y + xy^2 + t^6y^3 + t^3x^2 + t^{-1}x < +t^3y^2 + tx + ty + 1$.