

8. Exercise Discrete Geometrie II

Deadline: 11.12.2025 (before the Exercise class)

Each answer should be sufficiently proven.

1. Exercise (Affine Transformation)

Show, for every projective transformation π of \mathbb{P}^n , that maps ideal points to ideal points there exists a linear transformation $A \in GL_n(\mathbb{C})$ and a vector $v \in \mathbb{C}^n$ such that $\pi(\iota(x)) = \iota(Ax + v)$

2. Exercise (Subspace dimension)

Let $P(V)$ be a projective space induced by a vector space V . For every set $S \subset V$ the set

$$T = \{\text{lin}\{x\} : x \in S \setminus \{0\}\}$$

is a subset of $P(V)$ and for the subspace $\text{lin}(S)$ generated by S , $P(\text{lin}(S))$ is a projective subspace which we denote by $\langle T \rangle$. Prove the dimension formula

$$\dim U + \dim W = \dim(\langle U \cup W \rangle) + \dim(U \cap W)$$

for two arbitrary projective subspaces U and W of $P(V)$.

3. Exercise (Puiseux series)

Show that the set of Puiseux series, $\mathbb{P}\{\{t\}\}$, forms a field.