Technical University Berlin
Faculty II, Institute for Mathematics
Secretariat MA 6–2, Erik Oor
Prof. Dr. Michael Joswig
Marcel Wack.

WS 2025

7. Exercise Discrete Geometrie II

Deadline: 04.12.2025 (before the Exercise class)

Each answer should be sufficiently proven.

Definition 1 (smooth)**.** *A point p on a (plane projective) curve C_f defined by a homogeneous polynomial $f \in \mathbb{C}[x_1, x_2, x_3]$ is called* smooth *if there exists an $i \in \{1, 2, 3\}$ such that*

$$\frac{\partial f}{\partial x_i}(p) \neq 0.$$

The point p is called a singular *otherwise. The multiplicity of a point p on C_f is defined as the smallest integer m such that there exists a partial derivative of order m which does not vanish at p.*

1. Exercise (Singularities)

Consider the the curve C_f defined by the homogeneous polynomial

$$f(x, y) = x^2(x^2 + y^2) - 2(y^2 - x^2)^2.$$

Find all singular points of C_f in $\mathbb{P}^2(\mathbb{C})$, as well as their multiplicities.

2. Exercise (Parametric curve)

Take $C = \{(t^2, t^3 + 1) \in \mathbb{C}^2 \mid t \in \mathbb{C}\}$. Show that C is an algebraic curve. What are the irreducible components of C? What are the singular points of C?

3. Exercise (Bezout)

Show that if a plane projective curve $C \subset \mathbb{P}^2(\mathbb{C})$ of degree d has strictly more than $\frac{d}{2}$ singular points on a line L, then C is reducible in the sense that L is a component of C.

4. Exercise (Cubic intersections)

Let f be a cubic, i.e., a homogeneous polynomial of degree 3 in $\mathbb{C}[x_1, x_2, x_3]$, without singular points. For which points p on C_f does there exist another non-singular cubic that intersects C_f only in p?