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WS 2025

5. Exercise Discrete Geometrie II

Deadline: 20.11.2025 (before the Exercise class)

Each answer should be sufficiently proven.

1. Exercise (Tropical line)

Definition 1. *A* tropical conic *is a tropical hypersurface in* $\mathbb{R}^3/\mathbb{R}\mathbb{1}$ *of a homogeneous tropical polynomial of degree two.*

What are the combinatorially distinct types of tropical conics in $\mathbb{R}^3/\mathbb{R}\mathbb{1}$? What is a good definition for *combinatorially distinct* in this context?

2. Exercise (Lattice polytopes)

Proof or disprove: Every lattice polytope has a unimodular triangulation.

3. Exercise (Integer Splits)

Let $S = \delta\Delta_2 \cap \mathbb{Z}^2$ the intersection of a 2-dimensional simplex with the integer lattice.

$$A = \{(x, y) \in \mathbb{Z}^2 \mid x, y \geq 0, x + y \leq \delta\}.$$

Let Σ be the triangulation you obtain by subdividing S along lines between $(0, 0)$ and $(i, \delta + 1 - i)$ for $1 \leq i \leq \delta$. For which δ do all triangles of Σ have the same amount of interior lattice points?