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WS 2025

4. Exercise Discrete Geometrie II

Deadline: 12.11.2025 (before the Exercise class)

Each answer should be sufficiently proven.

1. Exercise (Tropical line)

Any tropical curve defined by a tropical polynomial of the form

$$F = a_1 \odot X_1 \oplus a_2 \odot X_2 \oplus c.$$

is called a *tropical line*. Determine all the possibilities for what a tropical line could look like. What about if a, b or c are ∞?

2. Exercise (Tropical projective torus)

Let $x \in \mathbb{R}^d$. Show that there exists exactly one $u \in x + \mathbb{R}\mathbb{1}$ with

1. $\min(u) = 0$, or
2. $\sum_{i=1}^{d} u_i = 0$ respectively.

These two properties can both be used to represent points in the tropical projective torus, $\mathbb{R}^d/\mathbb{R}\mathbb{1}$. What is the image of a max or min tropical polynomial of degree 1 in this representation?

3. Exercise (Smooth tropical curves)

A tropical curve is called *smooth* if its dual subdivision consists only of triangles of area $\frac{1}{2}$. Let $F : \mathbb{R}^2 \to \mathbb{R}$ be a tropical polynomial of degree d such that the tropical curve $\mathscr{T}(F)$ is smooth. Determine the number of vertices, unbounded edges and bounded edges of $\mathscr{T}(F)$.

4. Exercise (Tropical Hypersurface)

Draw the tropical hypersurface defined by the homogeneous tropical polynomial

$$(4 \odot X_1^3) \oplus (1 \odot X_1X_2X_3) \oplus (4 \odot X_2^3) \oplus (1 \odot X_2^2X_3) \oplus (1 \odot X_2X_3^2) \oplus (6 \odot X_3^3).$$

What does the dual subdivision of the Newton polytope look like?