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WS 2025

2. Exercise Discrete Geometrie II

Deadline: 29.10.2025 (before the Exercise class)

Each answer should be sufficiently proven.

1. Exercise (Restricting a triangulation)

Let Q and P be lattice polytopes such that $Q \subseteq P$. Show that there exists a triangulation of P that restricts to a triangulation of Q .

2. Exercise (Triangulations of polygons)

Let P be a polygon, i.e. a 2-dimensional polytope. Show, that every triangulation of P is regular.

3. Exercise (Lattice simplex)

Let $A = \delta\Delta_2 \cap \mathbb{Z}^2$ the intersection of a 2-dimensional simplex with the integer lattice.

$$A = \{(x, y) \in \mathbb{Z}^2 \mid x, y \geq 0, x + y \leq \delta\}.$$

Compute all $\delta \in \mathbb{N}$ for which A has a non-regular triangulation and give an example of such a triangulation.

4. Exercise (Volume)

Let $P := \Delta_{n-1} \times \Delta_1$ be a prism over the standard $(n-1)$ -simplex Δ_{n-1} , for $n \in \mathbb{N}_{\geq 2}$, i.e.

$$P = \{(x, y) \in \mathbb{R}^n \mid x \in \Delta_{n-1}, y \in \Delta_1\}.$$

- (a) Compute a triangulation of P .
- (b) Compute the volume of P .
- (c) Can you find all triangulations of P ?