ALGORITHMIC DISCRETE MATHEMATICS III: EXERCISES 3

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Exercise 1. Show that the group $\mathrm{PGL}_3(\mathbb{R})$ acts transitively on the set of (not necessarily convex) quadrangles, i.e., quadruples of points in the projective plane $\mathrm{PG}_2(\mathbb{R})$ such that no three are collinear. What are the $\mathrm{PGL}_3(\mathbb{R})$ orbits on the set of pentagons?

Exercise 2. Construct a triangulation of $\mathrm{PG}_2(\mathbb{R})$. That is, construct a finite simplicial complex which is homeomorphic to $\mathrm{PG}_2(\mathbb{R})$.

Exercise 3. Consider the 3-dimensional polyhedron P given by the following six linear inequalities:

$$\begin{aligned}
-2x_1 - 25x_2 + 10x_3 &\geq -25 \\
25x_1 + 2x_2 + 10x_3 &\geq 2 \\
-2x_1 + 25x_2 + 10x_3 &\geq -25 \\
25x_1 - 2x_2 + 10x_3 &\geq 2 \\
-x_2 - x_3 &\geq 0 \\
-x_2 + x_3 &\geq -2 \ .
\end{aligned}$$

(1) Show that P is bounded, i.e., a polytope, whose vertices are the eight columns of the matrix

$$\begin{pmatrix}
290/359 & 370/899 & 2/25 & 22/25 & 5/2 & 25/2 & -170/359 & 830/899 \\
81/359 & 621/899 & 0 & 0 & 0 & 0 & -621/359 & -81/899 \\
-637/359 & -621/899 & 0 & -2 & -2 & 0 & 621/359 & -1879/899
\end{pmatrix} .$$

(2) Check whether or not P is affinely/projectively/combinatorially equivalent to the 3-cube $[0,1]^3$.

Exercise 4. What is the maximal length of 3-dimensional spindle? Or, equivalently, what is the maximal width of a 3-dimensional prismatoid?

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