ALGORITHMIC DISCRETE MATHEMATICS III: EXERCISES 2

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The *moment map* is defined as $\mu : \mathbb{R} \to \mathbb{R}^n$, $t \mapsto (t, t^2, \dots, t^n)$. For $t_1 < t_2 < \dots < t_k$ and $k \geq n+1$ this gives rise to the *cyclic polytope*

$$C_{k,n} \ := \ \operatorname{conv}\bigl\{\mu(t_1), \mu(t_2), \dots, \mu(t_k)\bigr\} \ .$$

Exercise 1. Show that $C_{k,n}$ is simplicial for all $k \geq n+1$.

Exercise 2.

(1) Show that the product of two simple polytopes is always simple.
(2) Are there simplicial polytopes P and Q such that their product $P \times Q$ is simplicial?
(3) What is the operation on polytopes dual to the product?

The *permutahedron* Π_n is the polytope

$$\Pi_n \ := \ \operatorname{conv}\{\, (\sigma(1), \sigma(2), \dots, \sigma(n)) \mid \sigma \text{ permutation of } [n]\} \ .$$

Exercise 3. Show that Π_n is a simple polytope of dimension $n-1$.

Exercise 4. For any polytope $P \subset \mathbb{R}^n$ and any pair of distinct vertices, v and w, there is an (admissible) projective transformation $T \in \mathrm{PGL}_{n+1}\mathbb{R}$ and a linear objective function $c \in \mathbb{R}^n$ such that $v' := T \cdot v$ is the unique minimal vertex of the polytope $T \cdot P$, with respect to c, and $w' := T \cdot w$ is the unique maximal vertex.

Exercise 5. Can you find a simple 4-polytope with nine facets such that the diameter of its graph equals the Hirsch bound $9 - 4 = 5$?

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