OPTIMIZATION AND TROPICAL GEOMETRY: EXERCISES AND PROBLEMS 6

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Exercise 1. Show that the Laplacian Δ of a finite graph with n nodes is symmetric of rank $n-1$, and that $\ker \Delta$ is spanned by $\mathbf{1}$.

Exercise 2. Show that the dimension $r(D)$ of a divisor $D \in \mathrm{Div}(G)$ only depends on the equivalence class of D, i.e., $r(D') = r(D)$ whenever $D' \sim D$.

Exercise 3. For divisors $D, D' \in \mathrm{Div}(G)$ with $r(D) \geq 0$ and $r(D') \geq 0$ show that $r(D + D') \geq r(D) + r(D')$.

Exercise 4. Construct a graph G and two divisors $D, D' \in \mathrm{Div}^0(G)$ such that $|D|$ contains exactly one element and $|D'|$ contains more than one.

Exercise 5. For each $g \geq 1$ construct a graph G and a divisor $D \in \mathrm{Div}(G)$ of degree $g-1$ such that the configuration D in the chip firing game on G is not winning.

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Problem 6. In [1] it is shown that the linear system of a divisor on a tropical curve forms a module over the tropical semirung; i.e., it is a tropical cone.

Fix a point configuration $P \in \mathbb{Z}^2$ and consider the set $\mathcal{C}(P)$ all tropical plane curves which are dual some regular subdivisions of P. Let $C, C' \in \mathcal{C}(P)$ be two such tropical curves which are equivalent, i.e., their dual subdivisions agree. Further let $D \in \mathrm{Div}(C)$ and $D' \in \mathrm{Div}(C')$ be divisors supported on the same set of vertices of C (or C'). How are the linear systems of D and D' are related? Concepts and ideas from [2] may be useful.

REFERENCES

1. Christian Haase, Gregg Musiker, and Josephine Yu, *Linear systems on tropical curves*, Math. Z. **270** (2012), no. 3-4, 1111–1140. MR 2892941
2. Michael Joswig, Marta Panizzut, and Bernd Sturmfels, *The Schläfli fan*, 2019, Preprint `arXiv:1905.11951`.

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