# THIRD QUESTION

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## 3. How large is a Gröbner basis?

*Problem* 1. Consider the set $G = \{y^5, x^3y^2, x^4\}$ in $\mathbb{Q}[x, y]$. Show that $G$ is a Gröbner basis with respect to the pure lex ordering with $x > y$.

*Problem* 2 (Möller & Mora 1984). Fix $d \geq 2$. In the polynomial ring $\mathbb{Q}[x_0, x_1, x_2]$ consider the ideal

$$\langle x_2^d ,\ x_0^{d-1}x_2 - x_1^d \rangle \ .$$

Show that the reduced degree lexicographic Gröbner basis of $I$ contains the monomial $x_1^{d^2}$.

*Problem* 3 (Decker & Lossen 2005). In the polynomial ring $\mathbb{Q}[x, y, z]$ compute a (degree reverse lexicographic) Gröbner basis for the ideal

$$\langle 3x^3y + x^3 + xy^3 + y^2z^2 \ ,\ 2x^3z - xy - xz^3 - y^4 - z^2 \ ,\ 2x^2yz - 2xy^2 + xz^2 - y^4 \rangle \ .$$

What about, say, a lexicographic Gröbner basis?

*Problem* 4 (Kotsireas 2001). In the polynomial ring $\mathbb{Q}[B, D, F, b, d, f]$ let $G$ be the reduced degree reverse lexicographic Gröbner basis of the ideal

$$\left\langle \begin{matrix} (b-d)(B-D) - 2F + 2 \,, & (b-d)(B+D-2F) + 2(B-D) \,, \\ (b-d)^2 - 2(b+d) + f + 1 \,, & B^2b^3 - 1 \,, \\ D^2d^3 - 1 \,, & F^2f^3 - 1 \end{matrix} \right\rangle \ .$$

a. What is the largest absolute value of a coefficient occurring in some polynomial in $G$?
b. How large is the transformation matrix (text output, size in bytes)?

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