

# On the supraconvergence of elliptic finite difference schemes

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## Abstract

This paper deals with the supraconvergence of elliptic finite difference schemes on variable grids for second order elliptic boundary value problems subject to Dirichlet boundary conditions in two-dimensional domains. The assumptions in this paper are less restrictive than those considered so far in the literature allowing also variable coefficients, mixed derivatives and polygonal domains. The nonequidistant grids we consider are more flexible than merely rectangular ones such that e.g. local grid refinements are covered.

The results also develop a close relation between supraconvergent finite difference schemes and piecewise linear finite element methods. It turns out that the finite difference equation is a certain nonstandard finite element scheme on triangular grids combined with a special form of quadrature. In extension to what is known for the standard finite element scheme, here also the gradient is shown to be convergent of second order, and so our result is also a superconvergence result for the underlying finite element method.

**Key words:** nonuniform grids, finite difference scheme, stability, supraconvergence, superconvergence

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