Stabile Hochzeiten wie und warum?

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Was sind stabile Hochzeiten?

Gegeben:

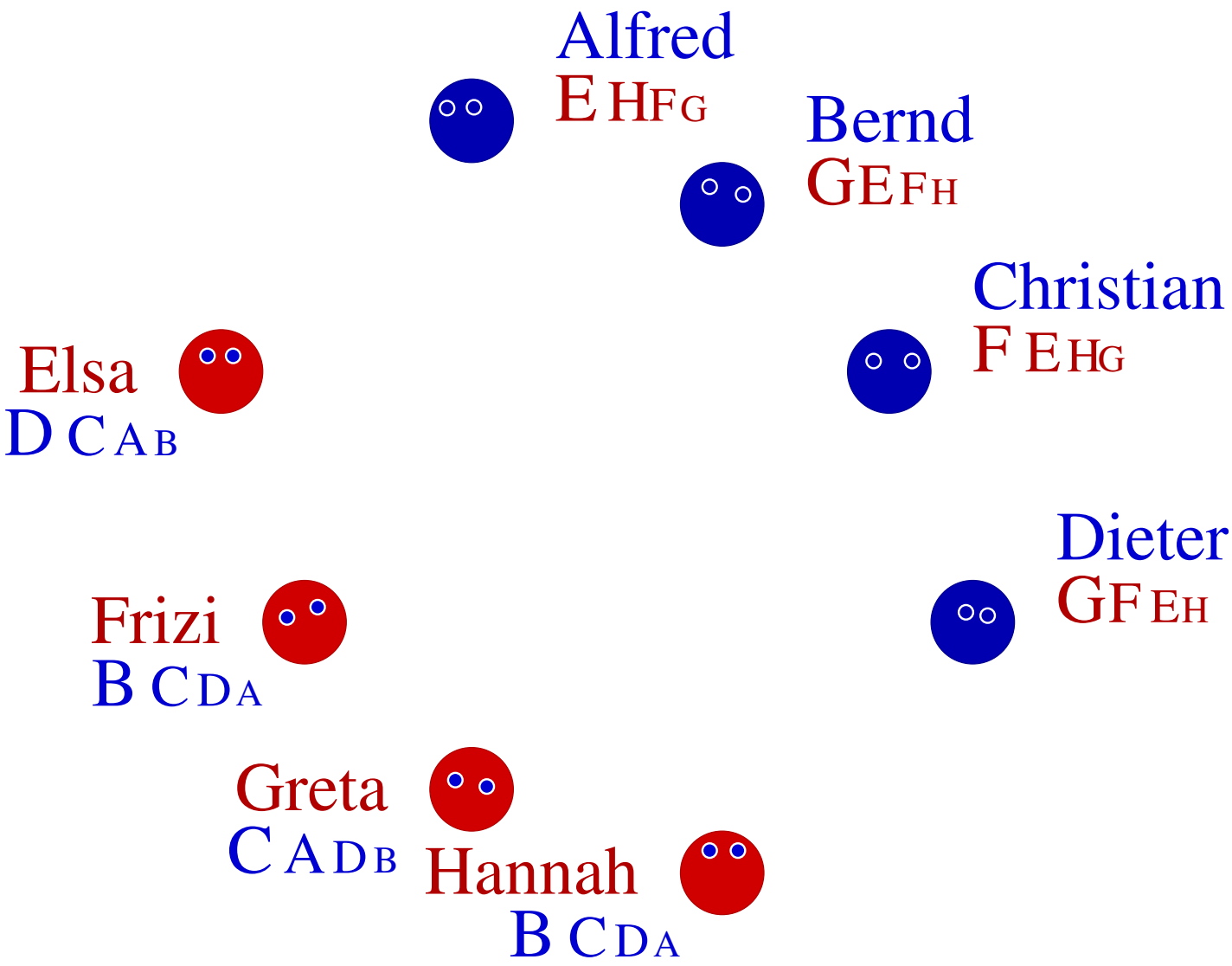
- F Menge von Frauen, M Menge von Männern, $|\mathsf{F}| = |\mathsf{M}|$.
- Jede Person hat Präferenzordnung auf Personen des anderen Geschlechts

Präferenz *Vorrang, Vorzug* (lat. *praeferre* «vorziehen»)

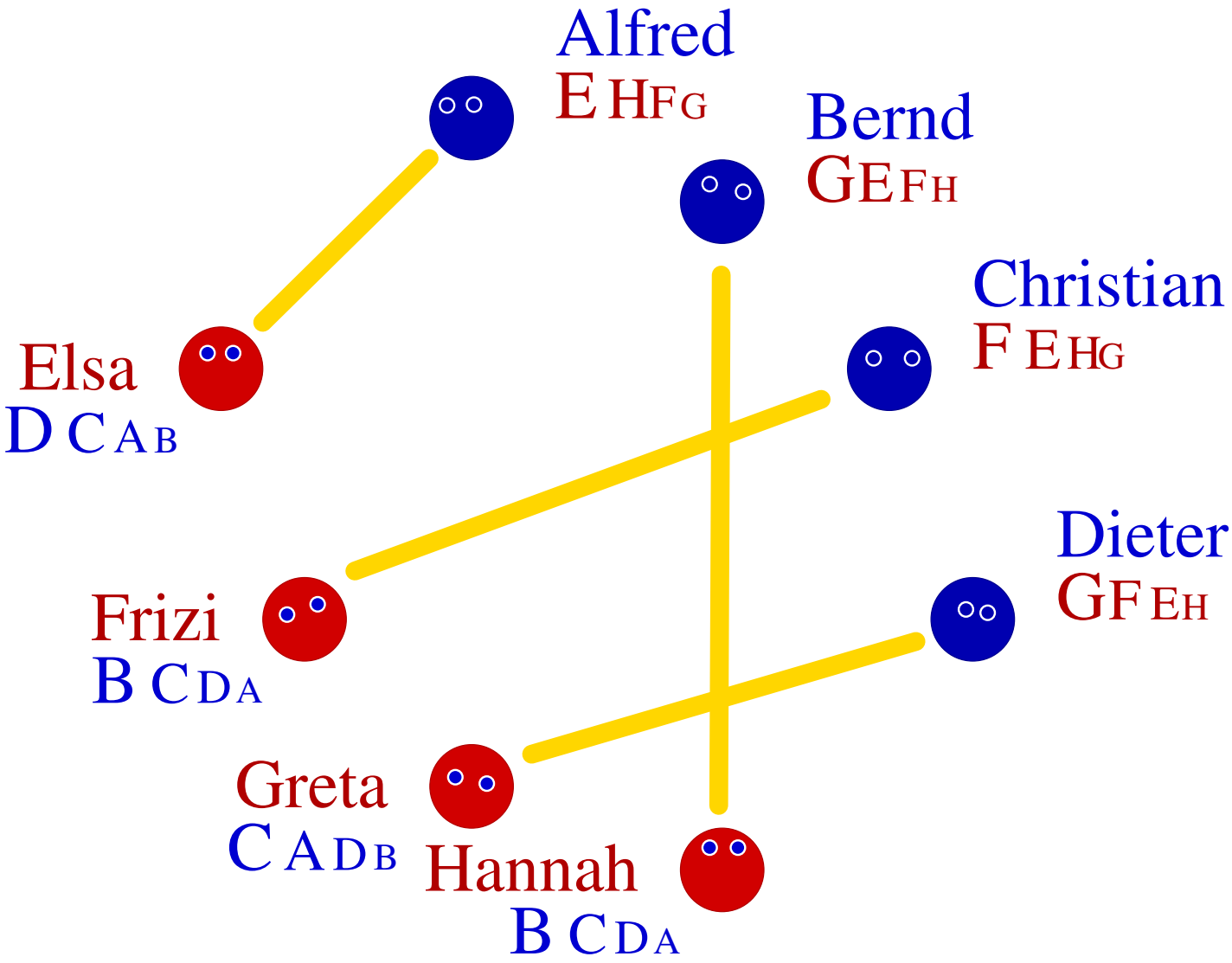
Ziel:

- Eine Massenhochzeit (Zuordnung) $\mathsf{M} \leftrightarrow \mathsf{F}$ die nicht ins Scheidungschaos führt (Stabilität).

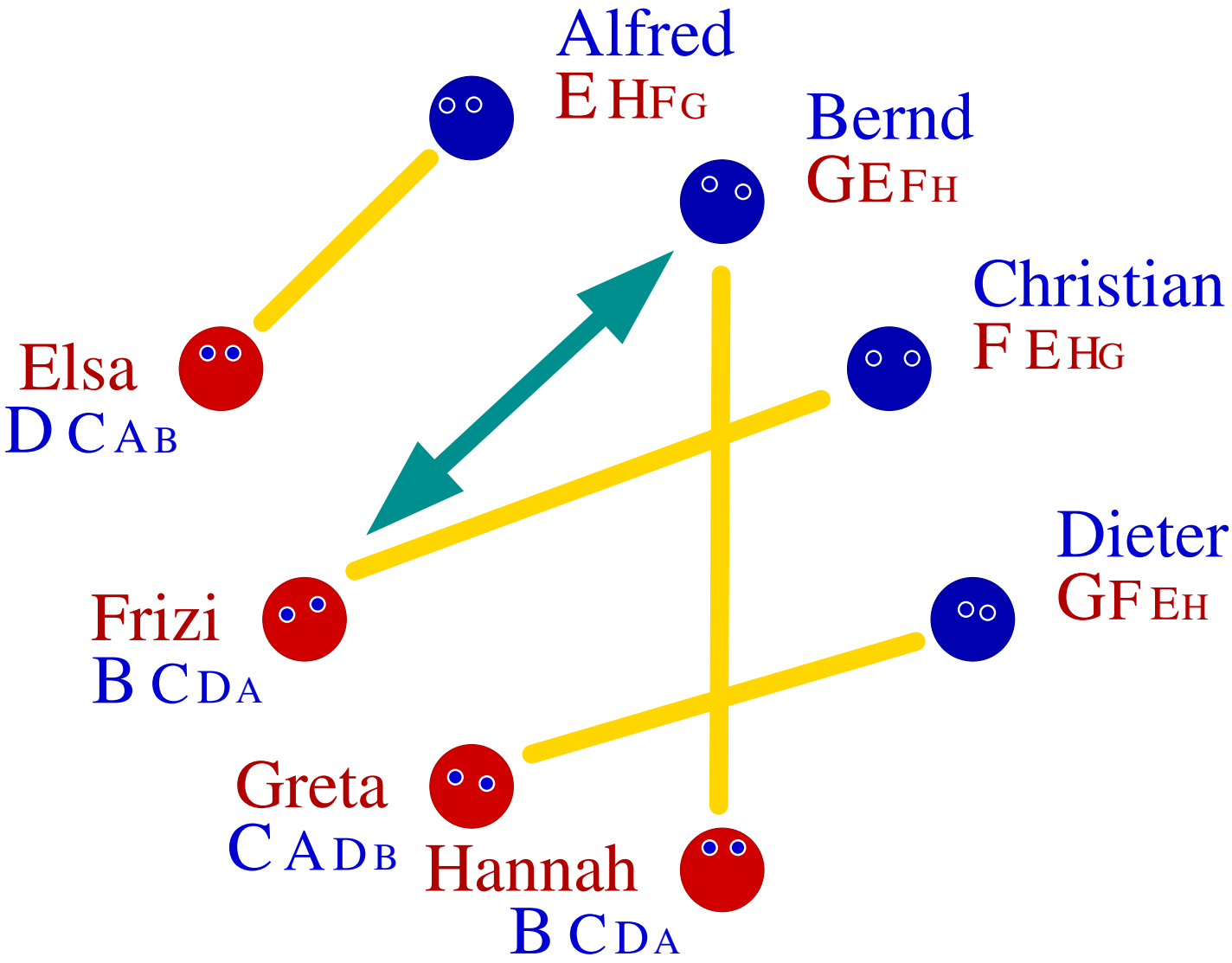
Präferenzlisten



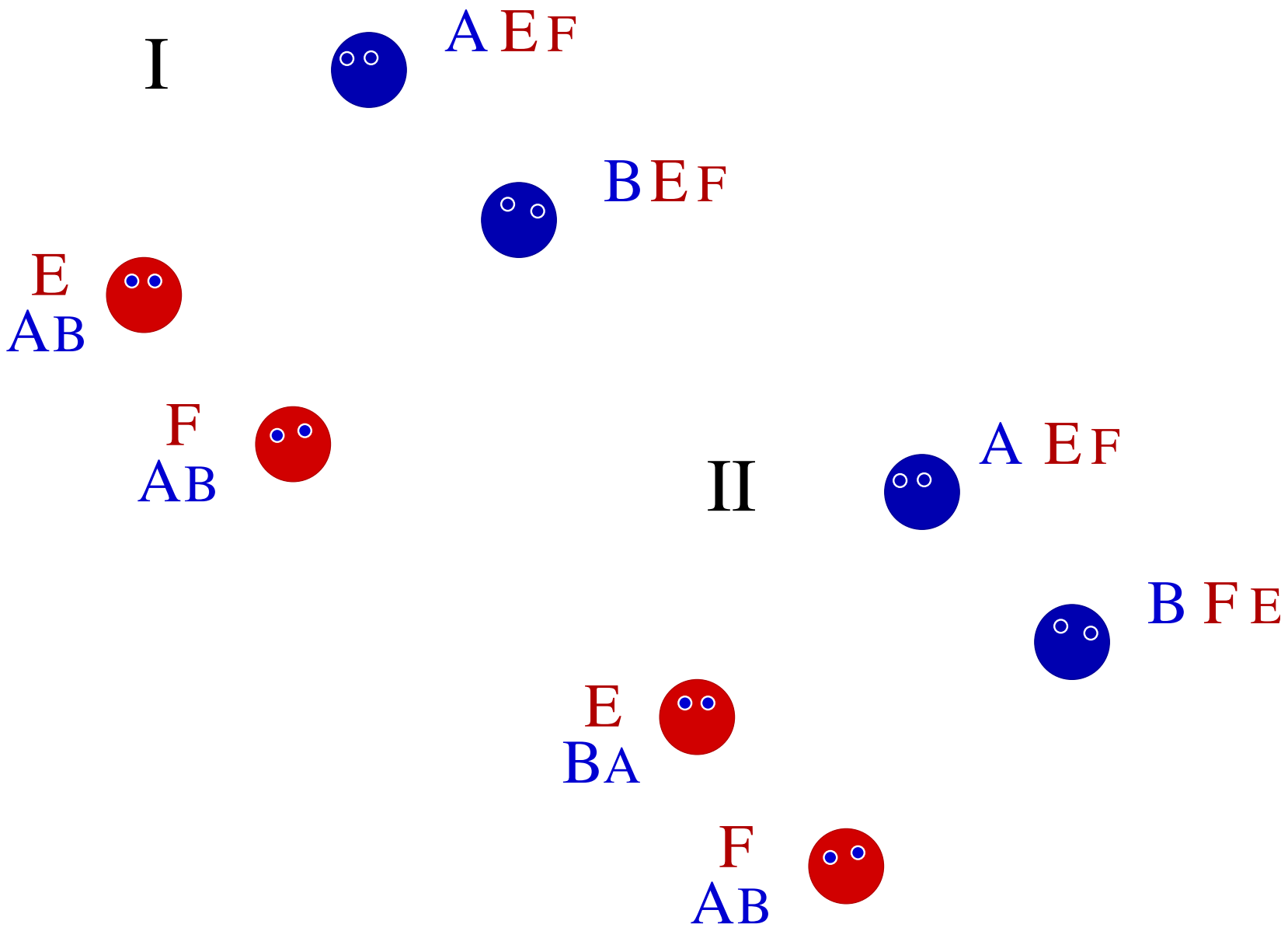
Eine Zuordnung „Hochzeit“



Eine Instabilität in der Hochzeit



Übung zum Thema Stabilität



Warum stabile Zuordnungen?

In vielen Situationen muss eine Zuordnung bestimmt werden

- Bewerber $\longleftrightarrow$ Plätze

Denkbar z.B. in Tanzschule oder bei Praktikumsbörse.

Tatsächlich werden stabile Zuordnungen angewandt:

- Frankreich: Wissenschaftler $\longleftrightarrow$ Uni-Stellen
- USA: Ältere Menschen $\longleftrightarrow$ Altenheime

Wie bekommt man stabile Zuordnungen?

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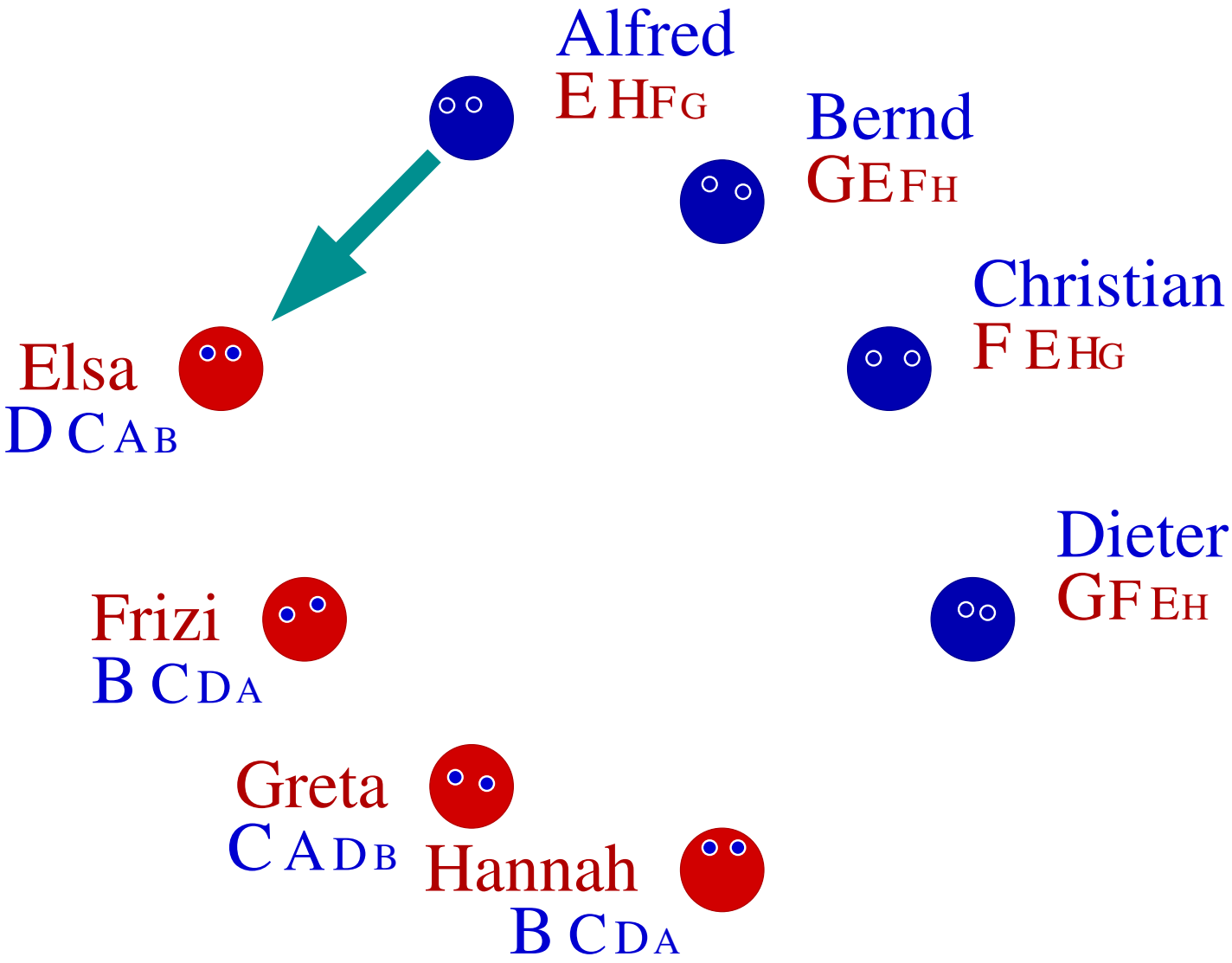
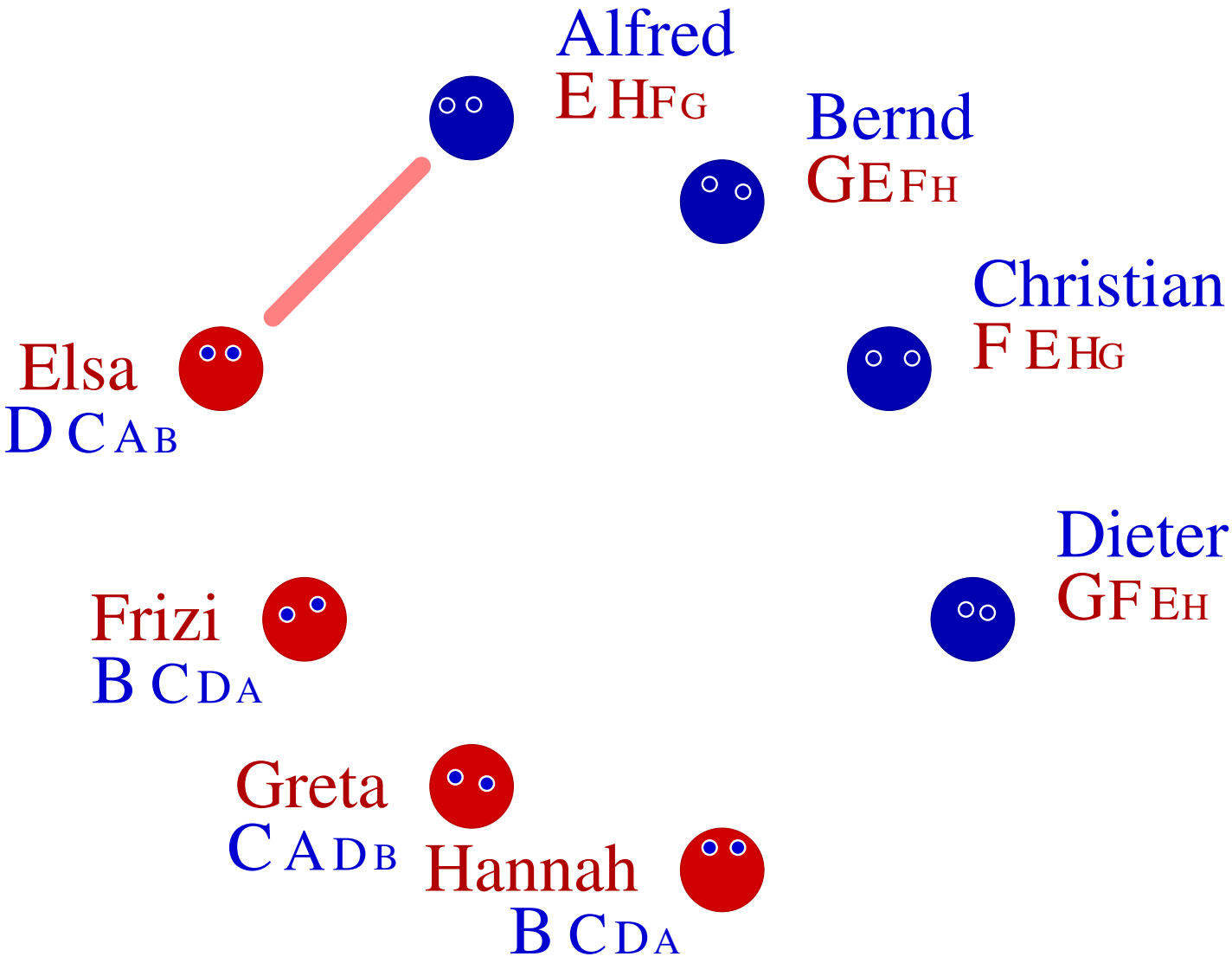
Die Grundidee:

Anträge und Verlobungen.

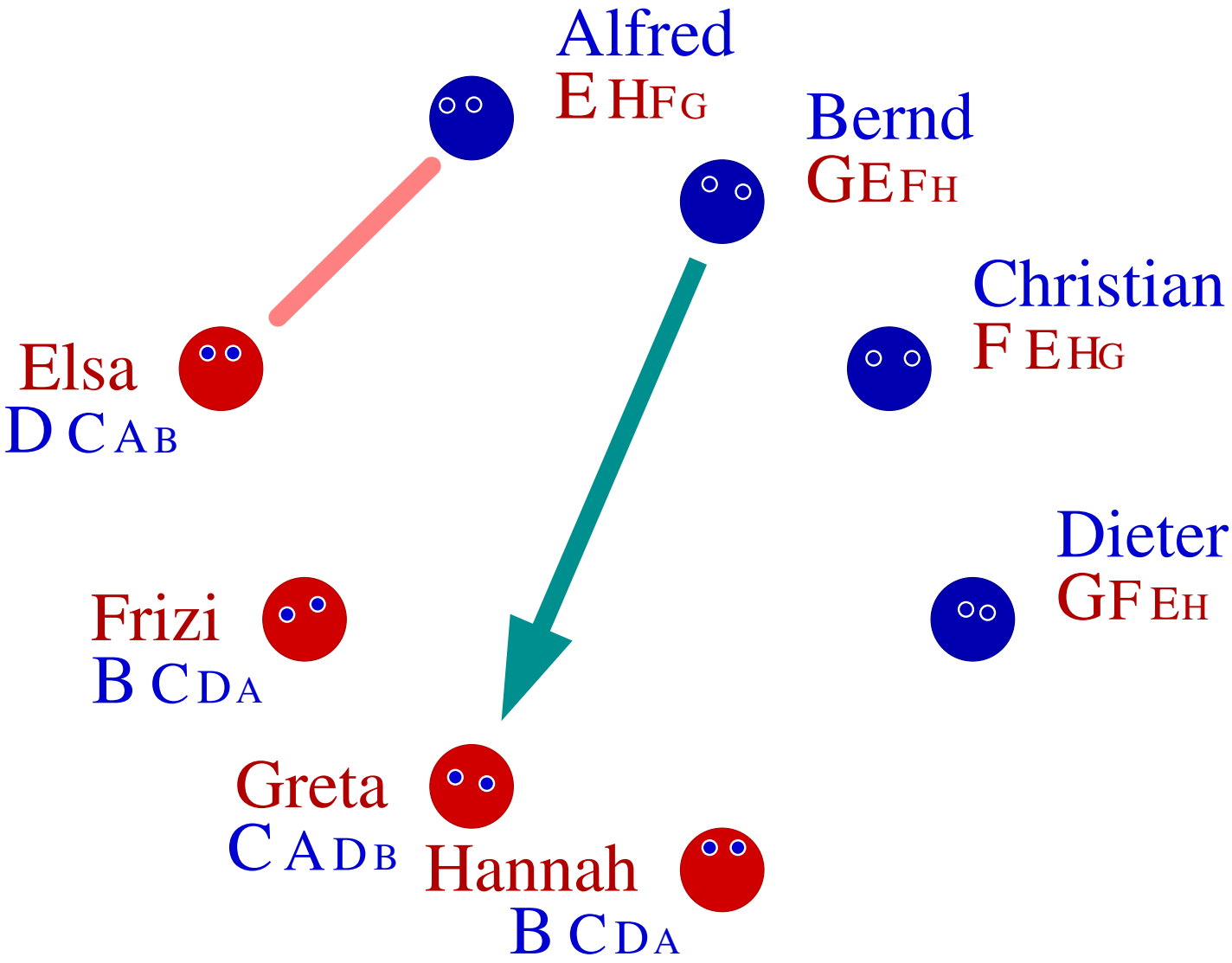
Ein Antrag von Alfred



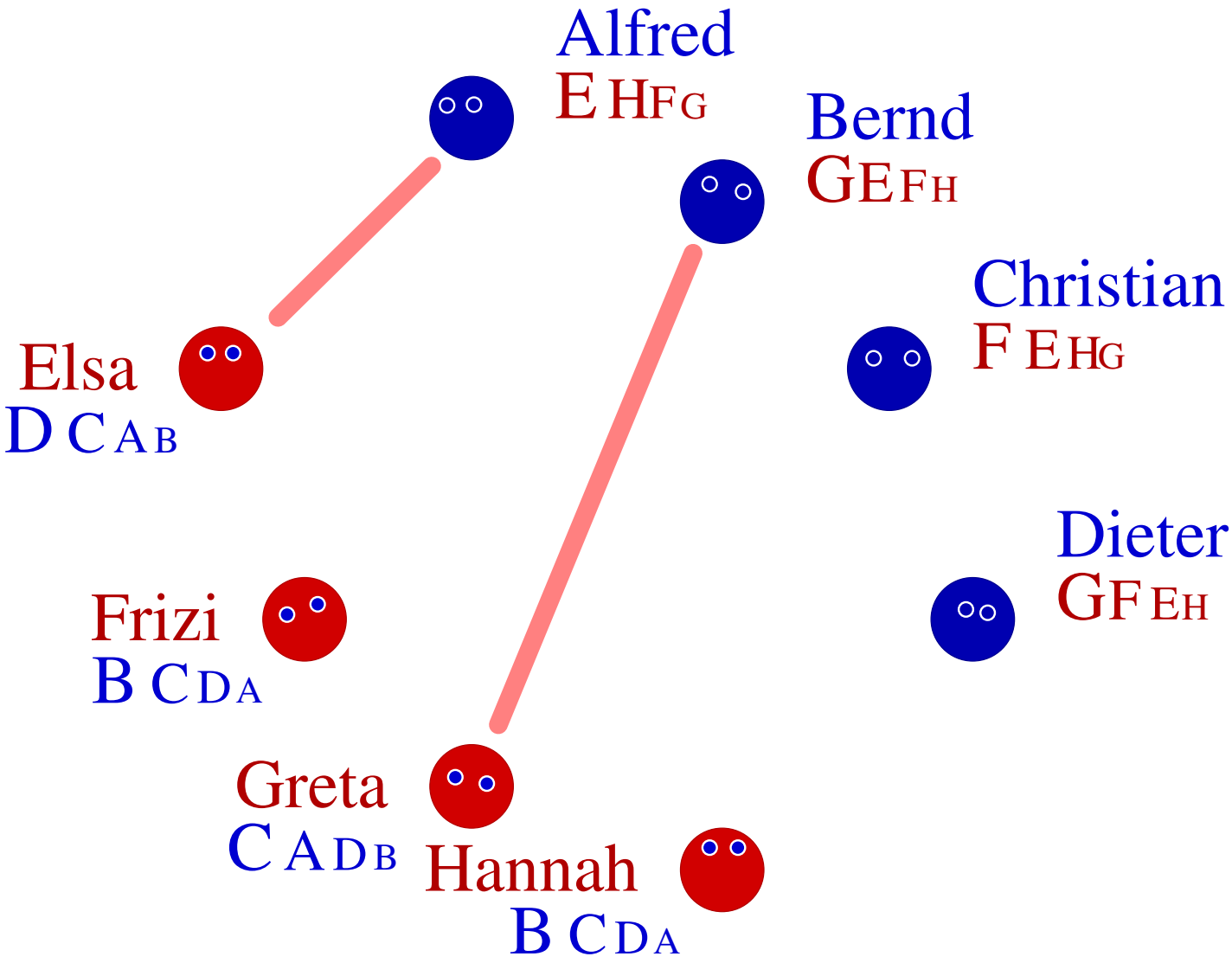
Verlobt: Alfred und Elsa



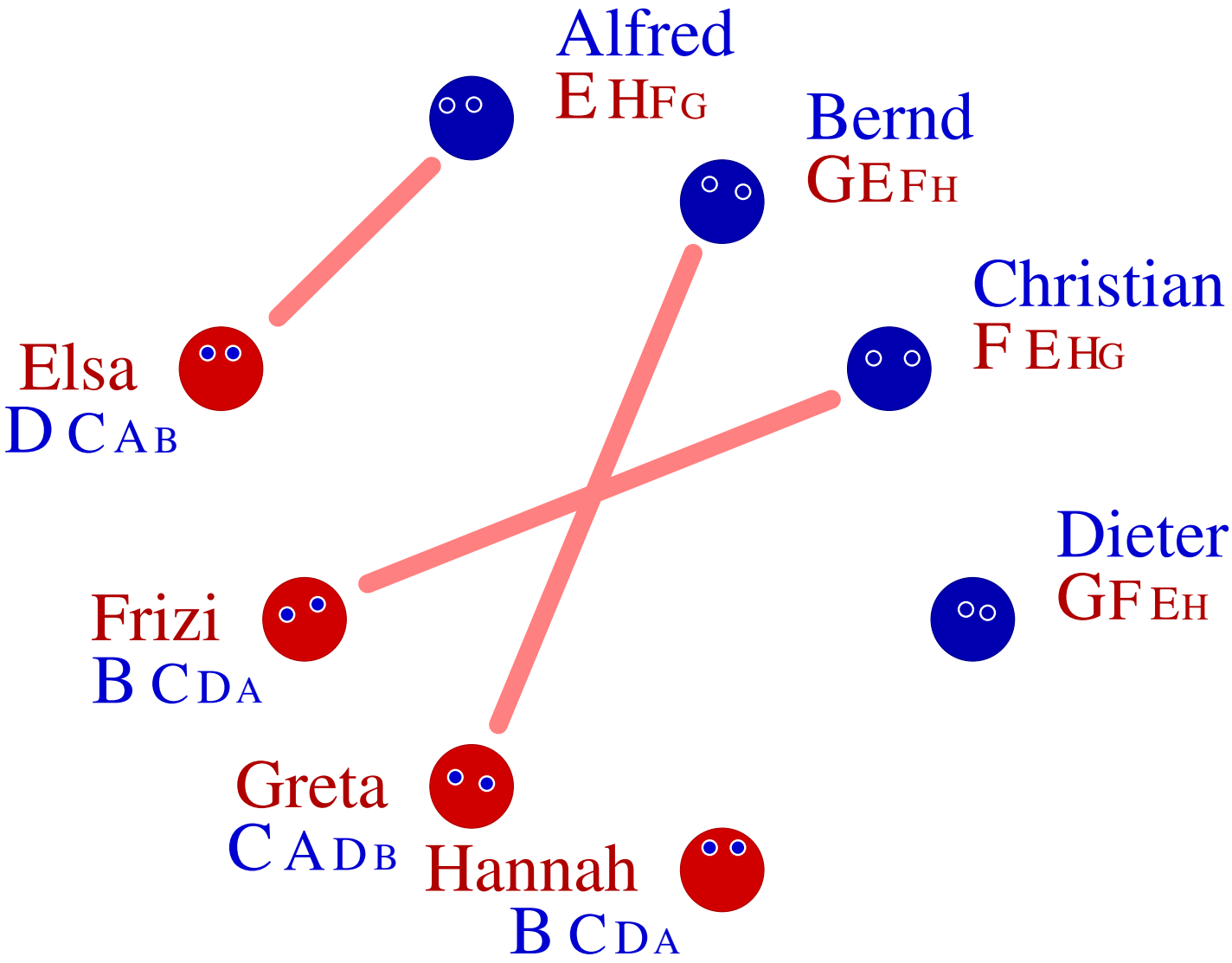
Ein Antrag von Bernd



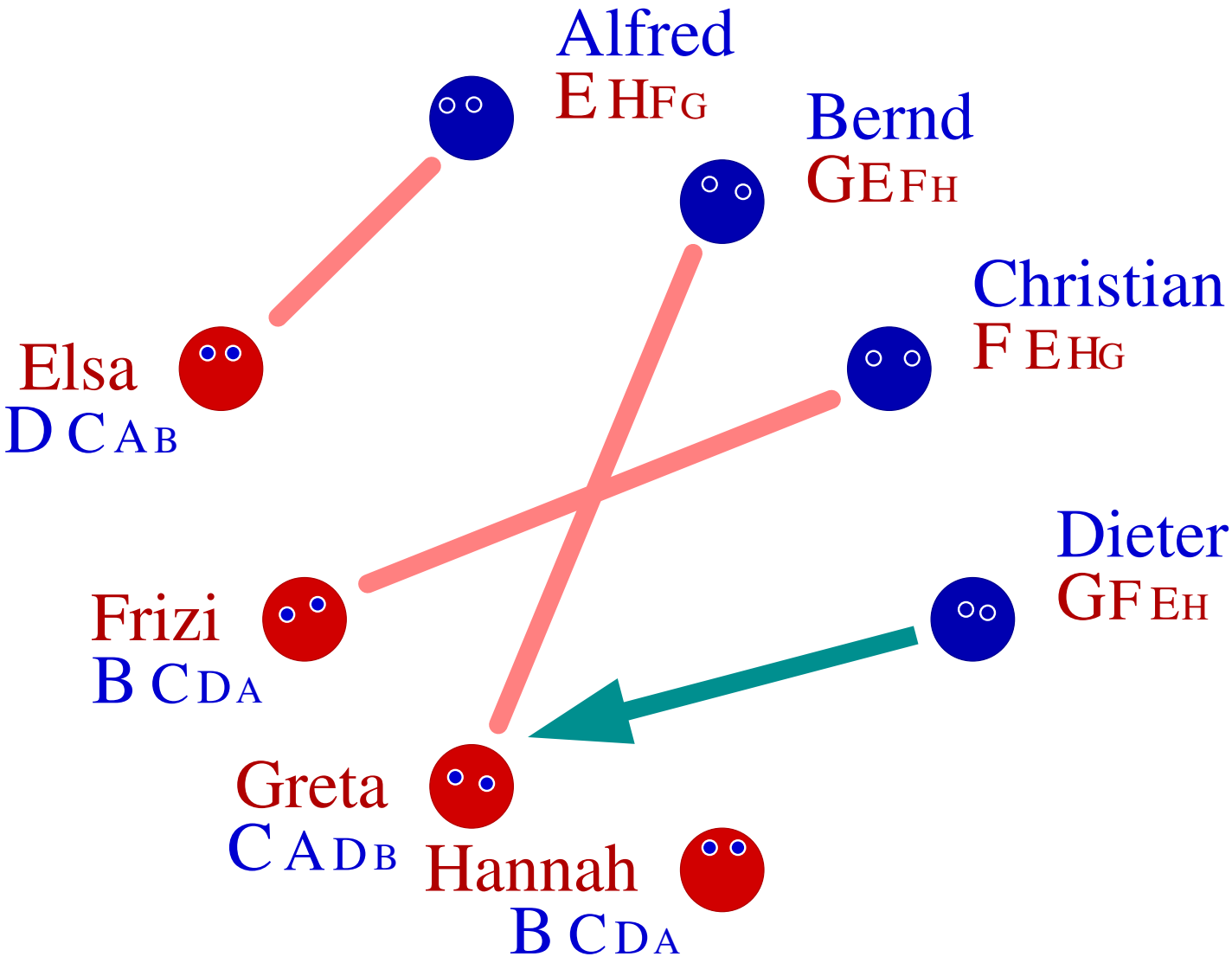
Verlobt: Bernd und Greta



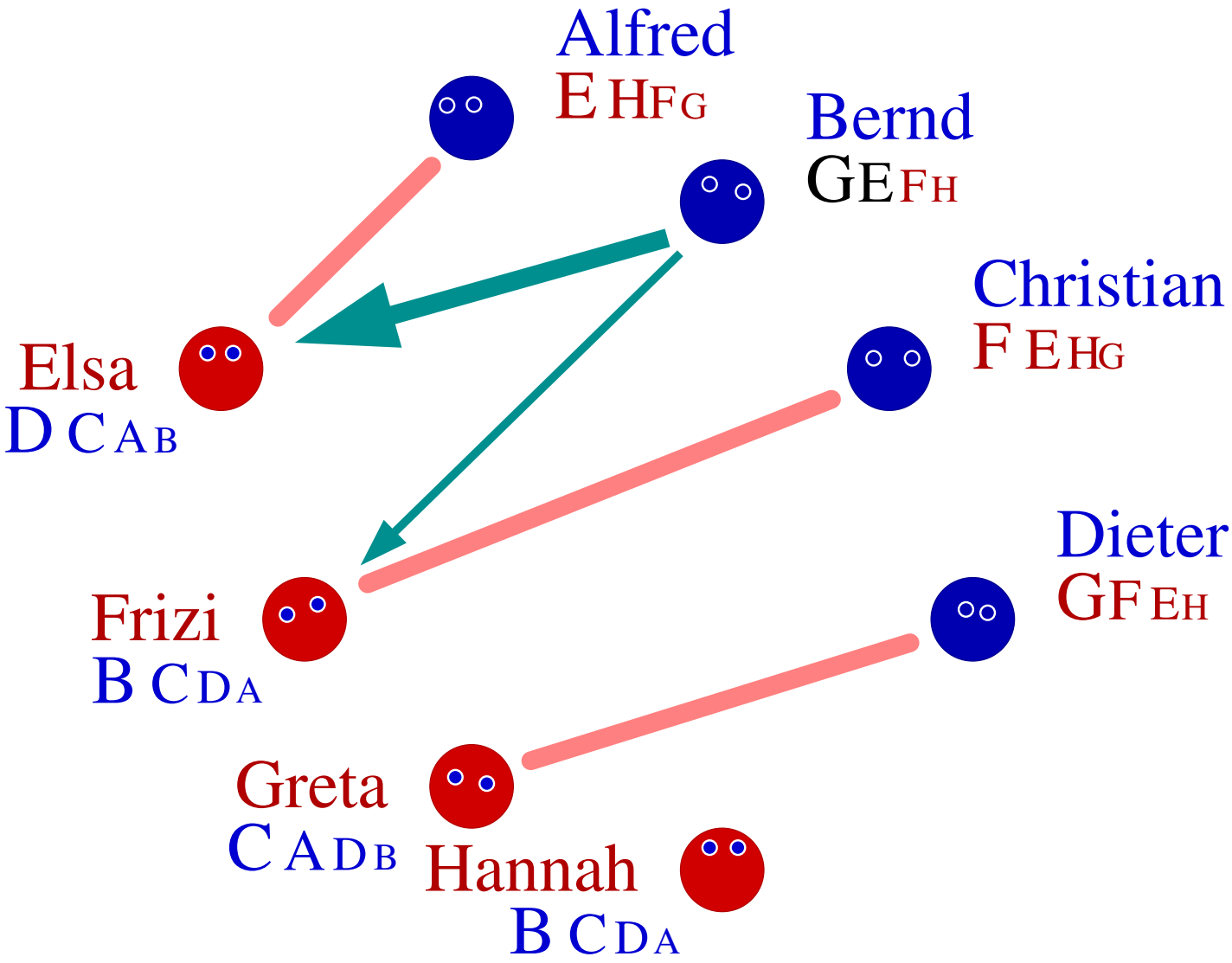
Antrag und verlobt: C & F



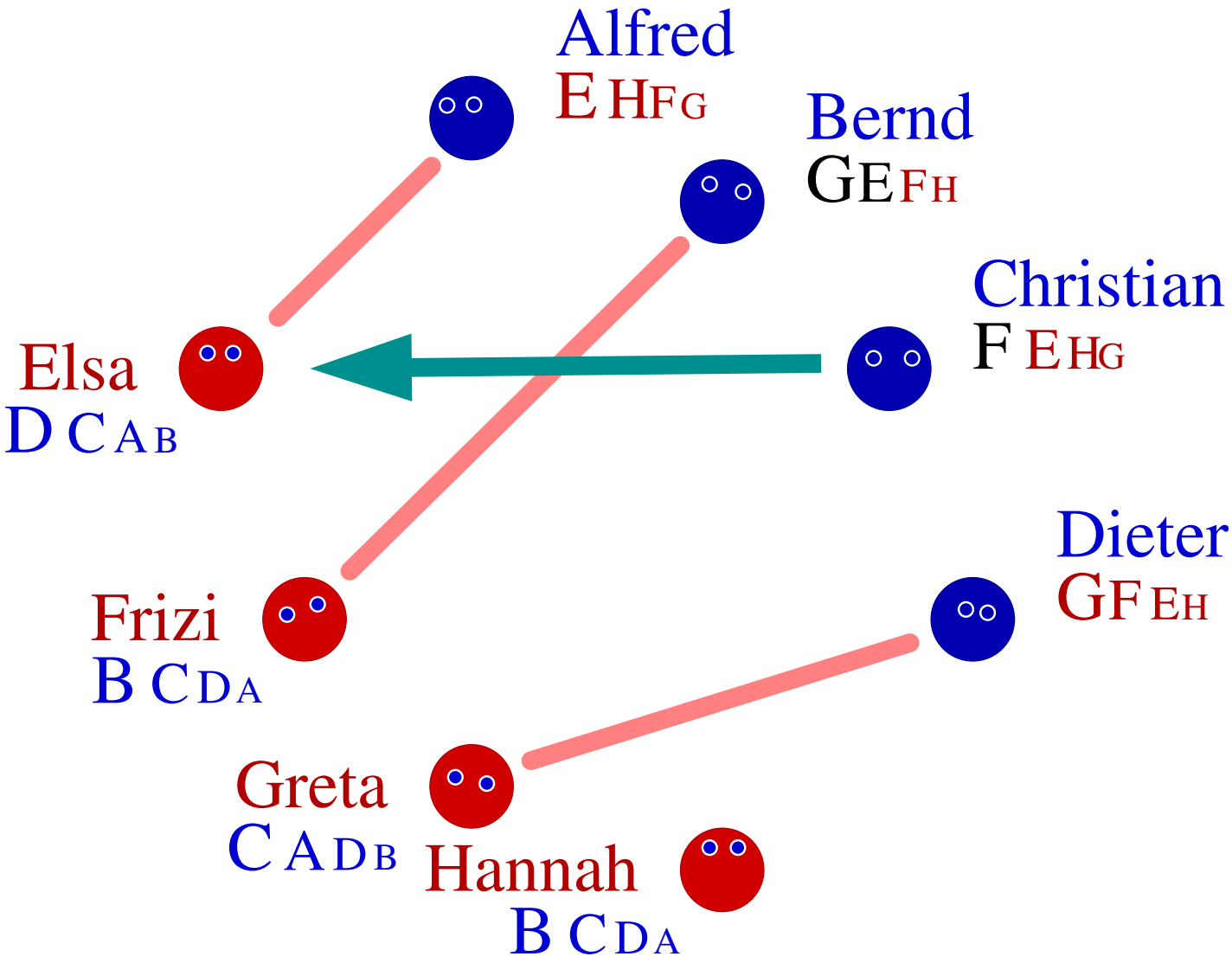
Antrag Dieter – Greta tauscht



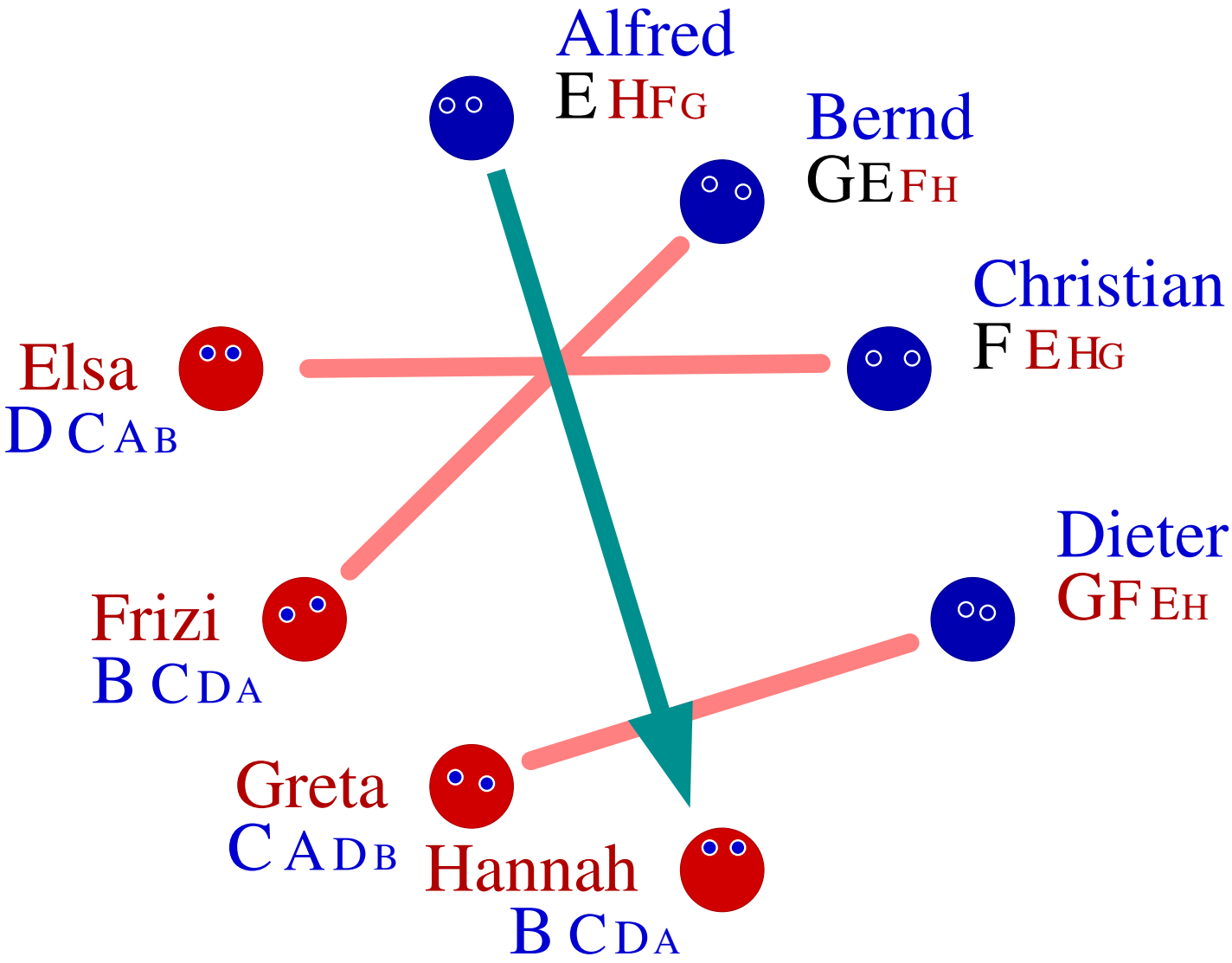
Anträge Bernd – Frizi tauscht



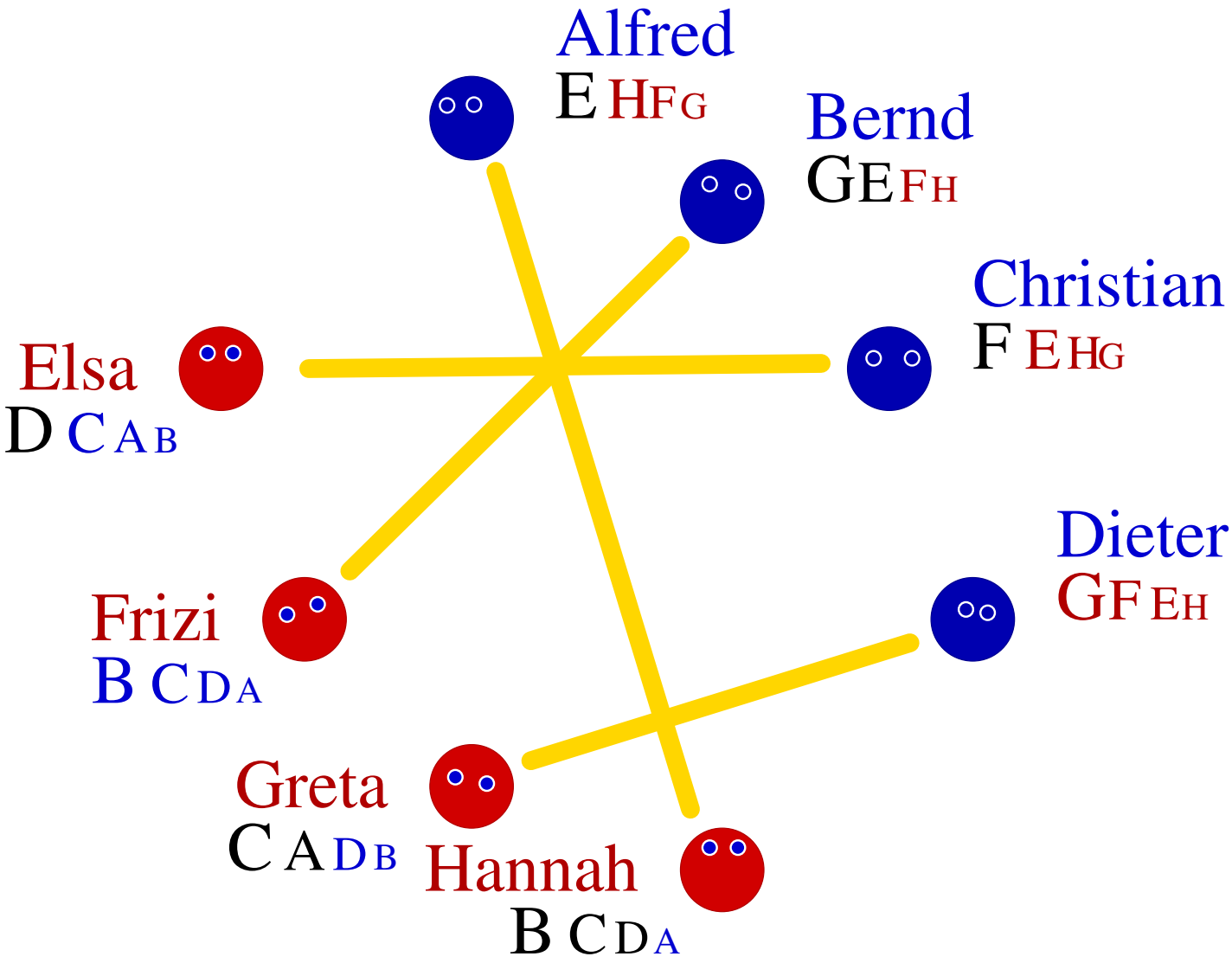
Antrag Christian – Elsa tauscht



Antrag Alfred – endlich Hannah



Alle verlobt, jetzt wir geheiratet



Der Hauptsatz

Satz [Gale & Shapley 1962]**.**
Die Antragsmethode führt immer zu einer stabilen Hochzeit.

COLLEGE ADMISSIONS AND THE STABILITY OF MARRIAGE

D. GALE* AND L. S. SHAPLEY, Brown University and the RAND Corporation

1. Introduction. The problem with which we shall be concerned relates to the following typical situation: A college is considering a set of n applicants of which it can admit a quota of only q. Having evaluated their qualifications, the admissions office must decide which ones to admit. The procedure of offering admission only to the q best-qualified applicants will not generally be satisfactory, for it cannot be assumed that all who are offered admission will accept. Accordingly, in order for a college to receive q acceptances, it will generally have to offer to admit more than q applicants. The problem of determining how many and which ones to admit requires some rather involved guesswork. It may not be known (a) whether a given applicant has also applied elsewhere; if this is known it may not be known (b) how he ranks the colleges to which he has applied; even if this is known it will not be known (c) which of the other colleges will offer to admit him. A result of all this uncertainty is that colleges can expect only that the entering class will come reasonably close in numbers to the desired quota, and be reasonably close to the attainable optimum in quality.

The usual admissions procedure presents problems for the applicants as well as the colleges. An applicant who is asked to list in his application all other colleges applied for in order of preference may feel, perhaps not without reason, that by telling a college it is, say, his third choice he will be hurting his chances of being admitted.

One elaboration is the introduction of the "waiting list," whereby an applicant can be informed that he is not admitted but may be admitted later if a vacancy occurs. This introduces new problems. Suppose an applicant is accepted by one college and placed on the waiting list of another that he prefers. Should he play safe by accepting the first or take a chance that the second will admit him later? Is it ethical to accept the first without informing the second and then withdraw his acceptance if the second later admits him?

We contend that the difficulties here described can be avoided. We shall describe a procedure for assigning applicants to colleges which should be satisfactory to both groups, which removes all uncertainties and which, assuming there are enough applicants, assigns to each college precisely its quota.

2. The assignment criteria. A set of n applicants is to be assigned among m colleges, where q_i is the quota of the ith college. Each applicant ranks the colleges in the order of his preference, omitting only those colleges which he would never accept under any circumstances. For convenience we assume there are no ties; thus, if an applicant is indifferent between two or more colleges he is nevertheless required to list them in some order. Each college similarly ranks the students who have applied to it in order of preference, having first eliminated those appli-

* The work of the first author was supported in part by the Office of Naval Research under Task NR047-018.

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D. Gale and L. S. Shapley: *College Admissions and the Stability of Marriage*, American Mathematical Monthly 69, 9-14, 1962

1224 Zitate.

Gale und Shapley

David Gale
(13.12.1921 – 7.3.2008)

Mathematikprofessor
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Puzzle Liebhaber
Spieleerfinder
Interaktives Mathe. Museum
http://mathsite.math.berkeley.edu

Gale transform

Lloyd Shapley
(2.6.1923 –)

Mathematikprofessor
Rand Corp., Princeton, UCLA

Spieltheorie

Shapley value

Zurück zur Theorie

- Die Antragsmethode führt immer zu einer stabilen Hochzeit

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- Egal in welcher Reihenfolge die Burschen ihre Anträge machen, das Ergebnis ist immer das gleiche.
- Die Verlobten der Burschen werden, in deren Präferenz, immer schlechter.
- Die Verlobten der Mädchen werden, in deren Präferenz, immer besser.

Die Theorie sagt ausserdem:

- Jeder Bursche heiratet das beste Mädchen das er in einer stabilen Hochzeit haben kann.
- Jedes Mädchen heiratet den übelsten Burschen den sie in einer stabilen Hochzeit haben kann.

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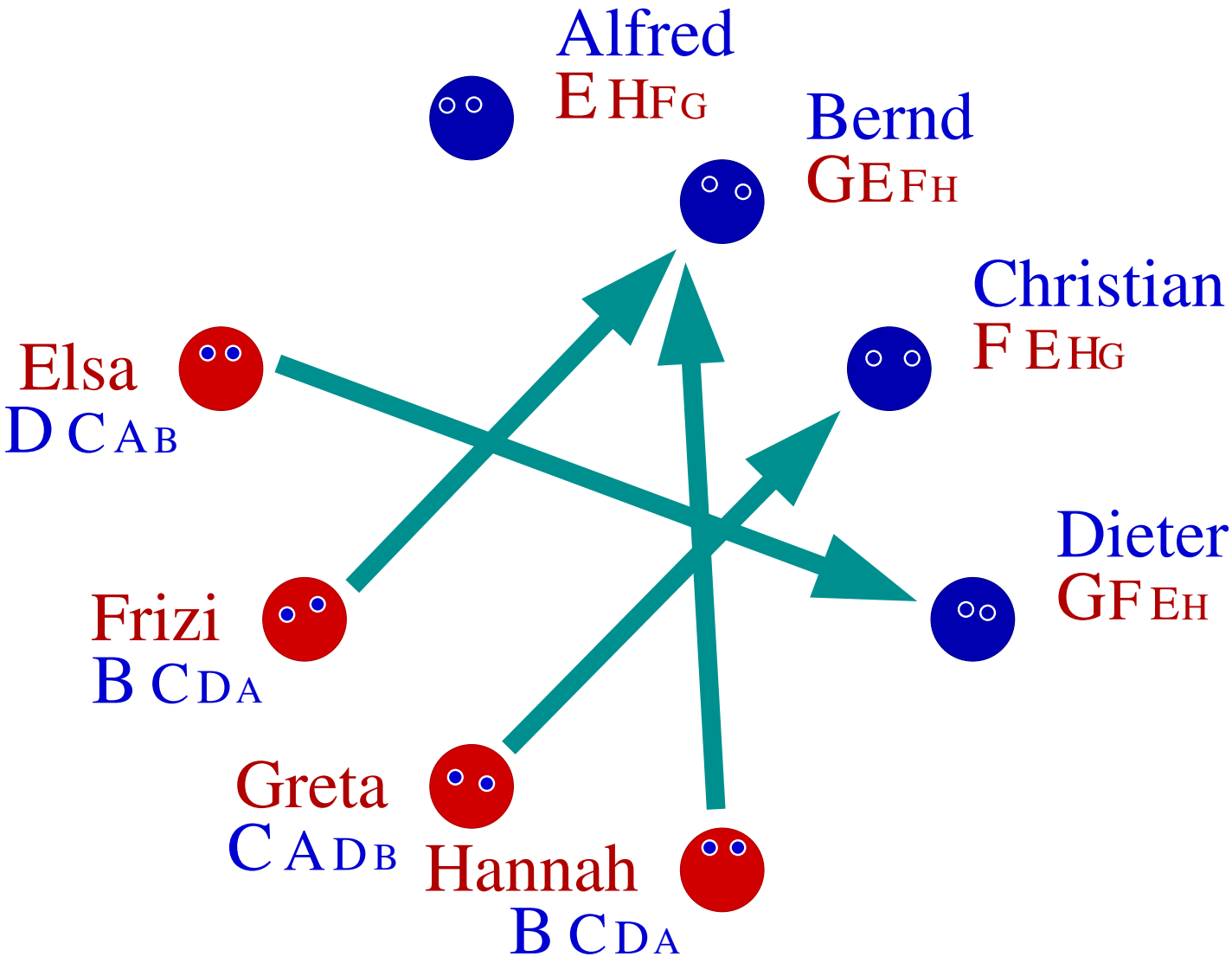
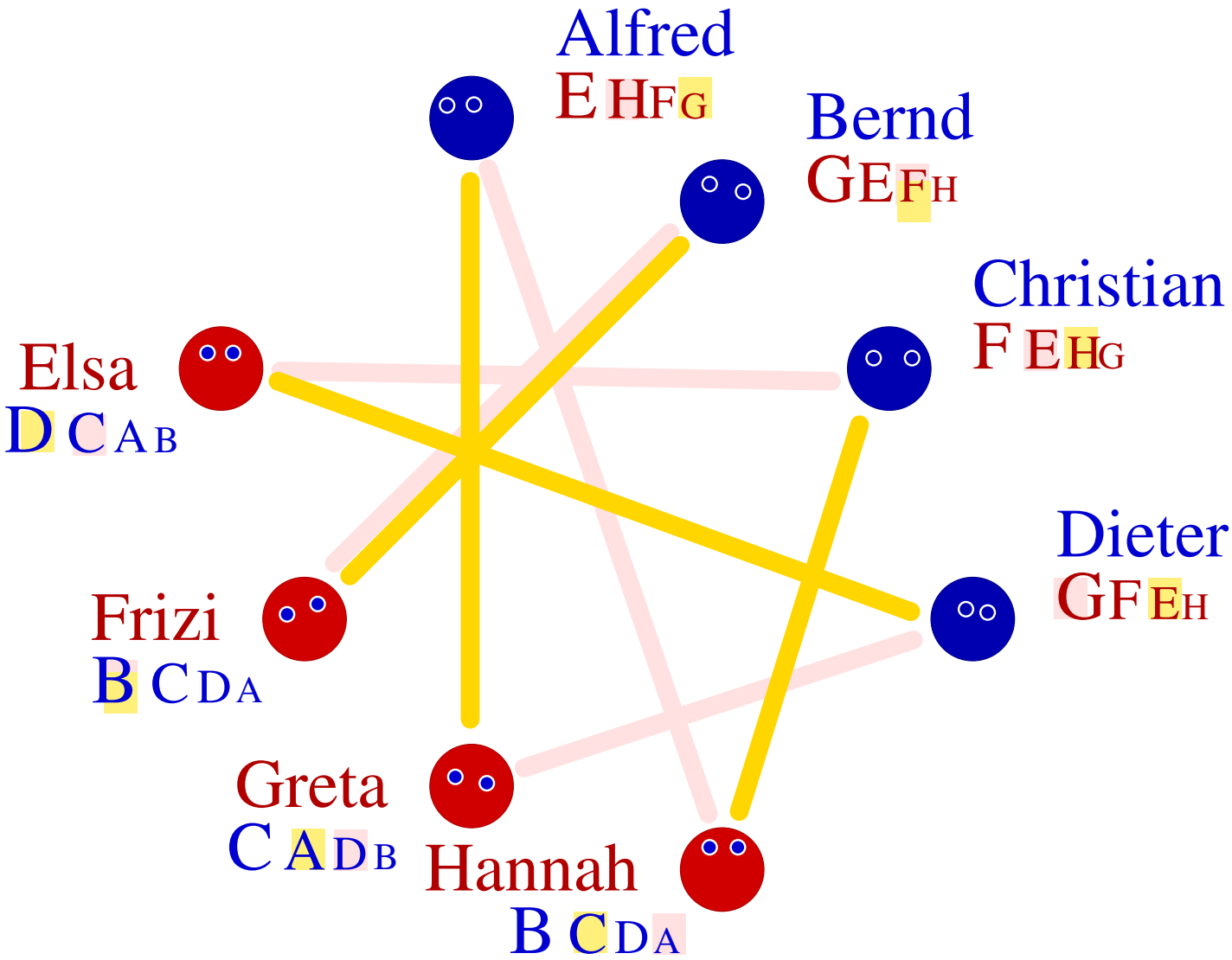
- Hannah sagt: Das ist Mist, jetzt werden wir aktiv!

Frauenanträge



Die zweite stabile Hochzeit



Die Lehre

- Aktivität ist anstrengend und oft frustrierend.
  Aber sie zahlt sich aus.

Die Mathematik - fragt weiter

- Gibt es weitere Struktur auf den stabilen Hochzeiten?
- Wie viele kann es geben?
- Kann man Stabilität in unisex Situationen erreichen? (Zimmerpartner Problem)
- Wie schnell kann man eine stabile Lösung berechnen?
- Welche Probleme machen partielle Listen?
- ....

Die Struktur

A:	a	b	c	d	e	f
B:	b	a	d	c	e	f
C:	c	d	a	b	e	f
D:	d	c	b	a	e	f
E:	e	f	a	b	c	d
F:	f	e	a	b	c	d

a:	D	C	B	A	F	E
b:	C	D	A	B	F	E
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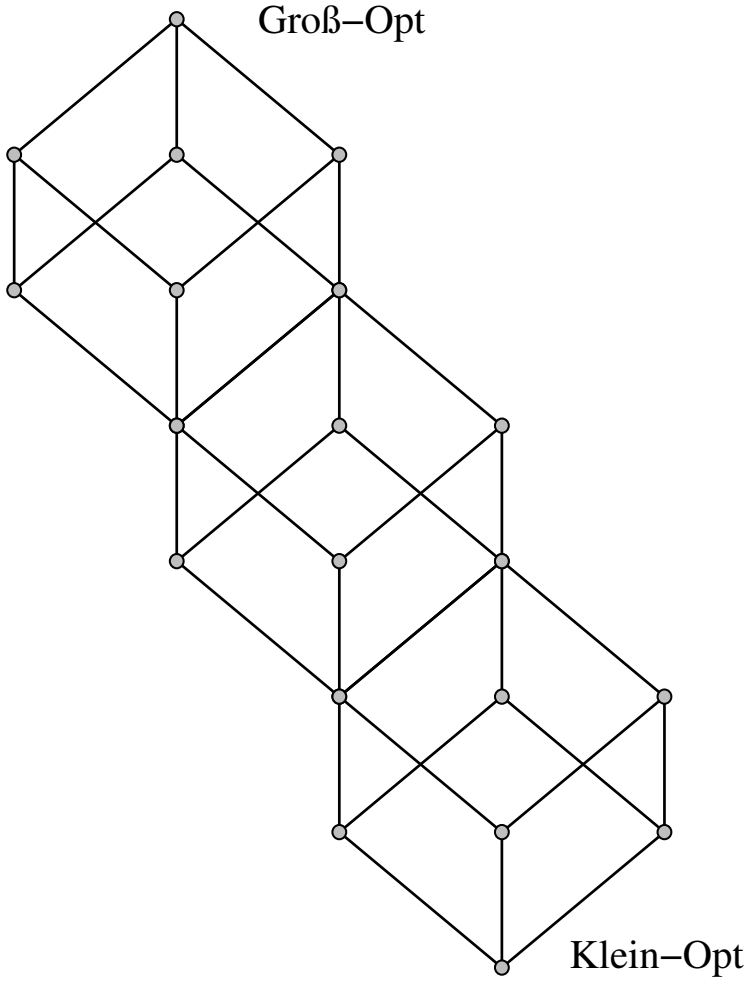
f:	E	F	D	C	B	A



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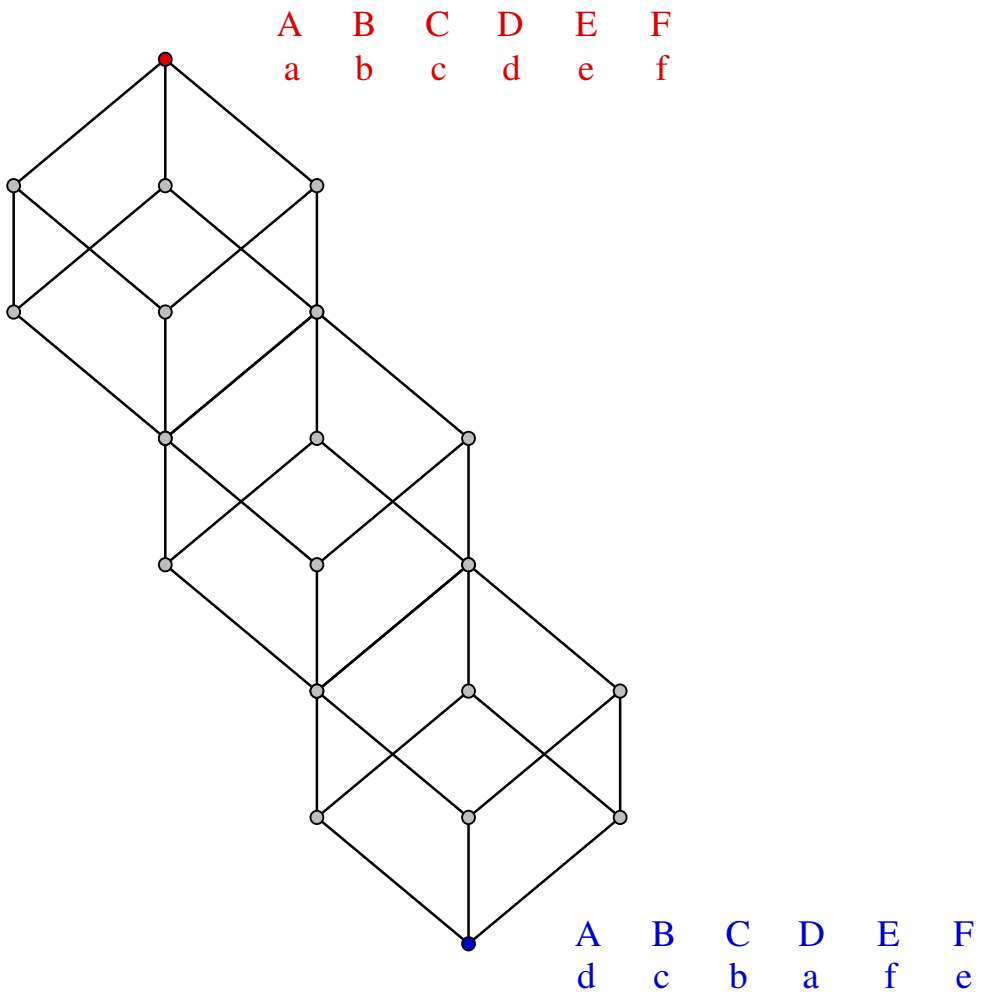

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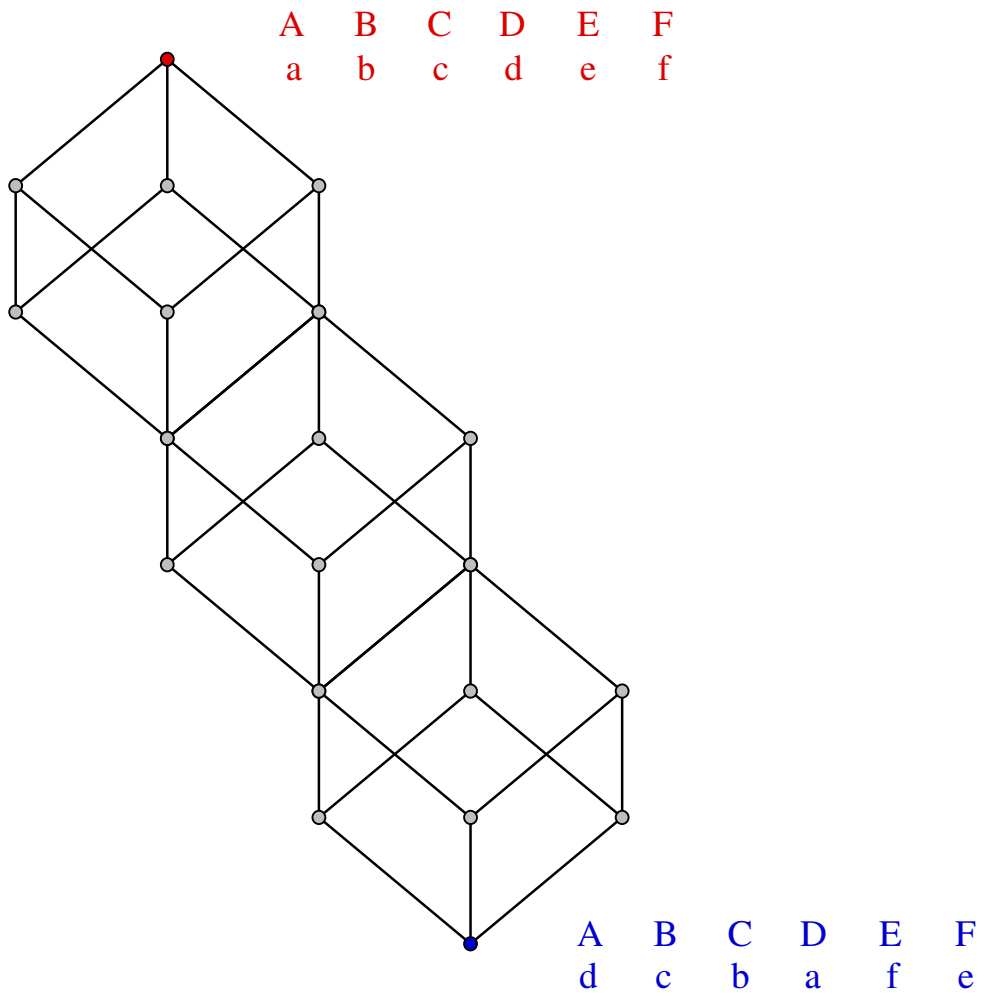
f:	E	F	D	C	B	A



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E:	e	f		
F:	f	e		

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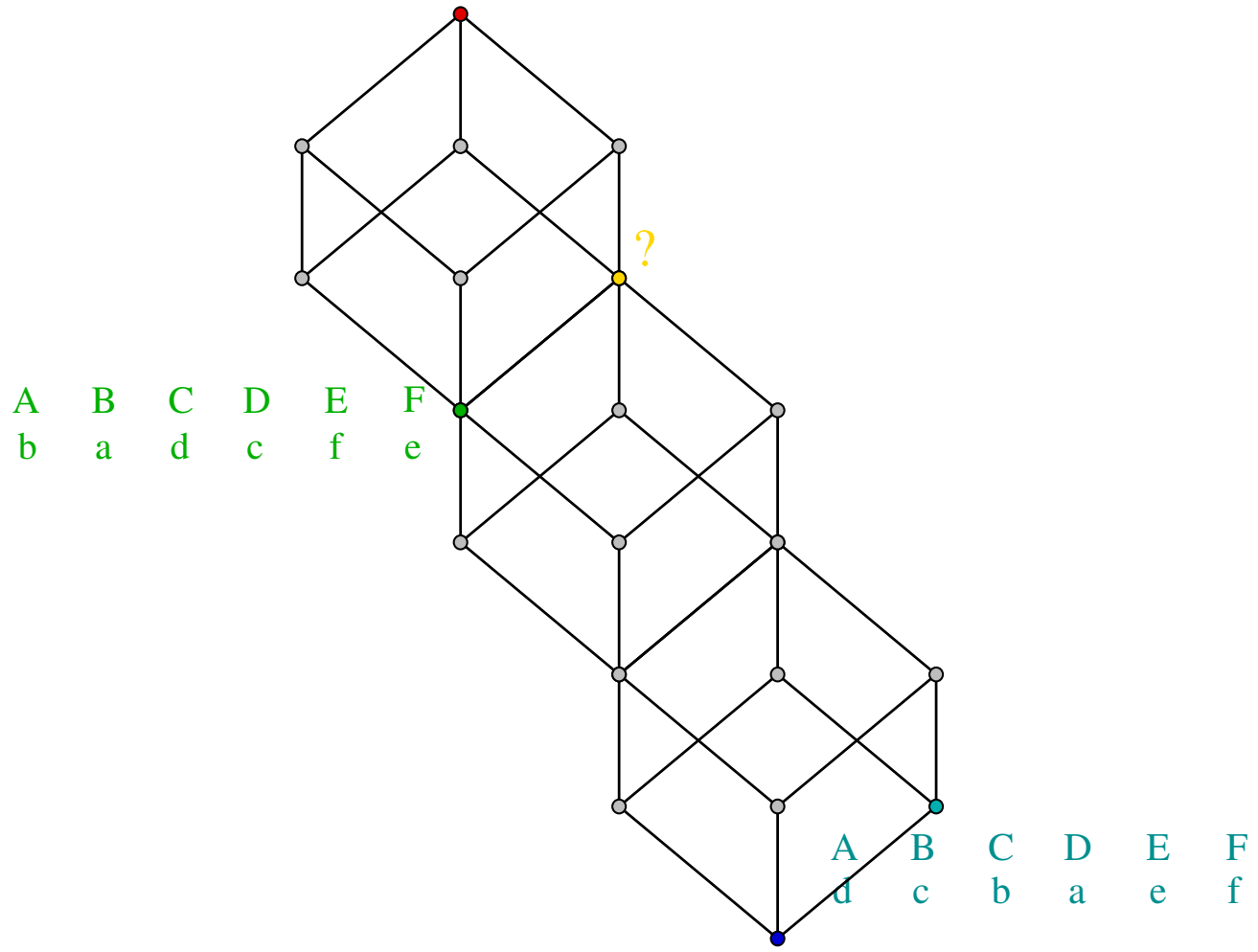
e:	F	E		
f:	E	F		



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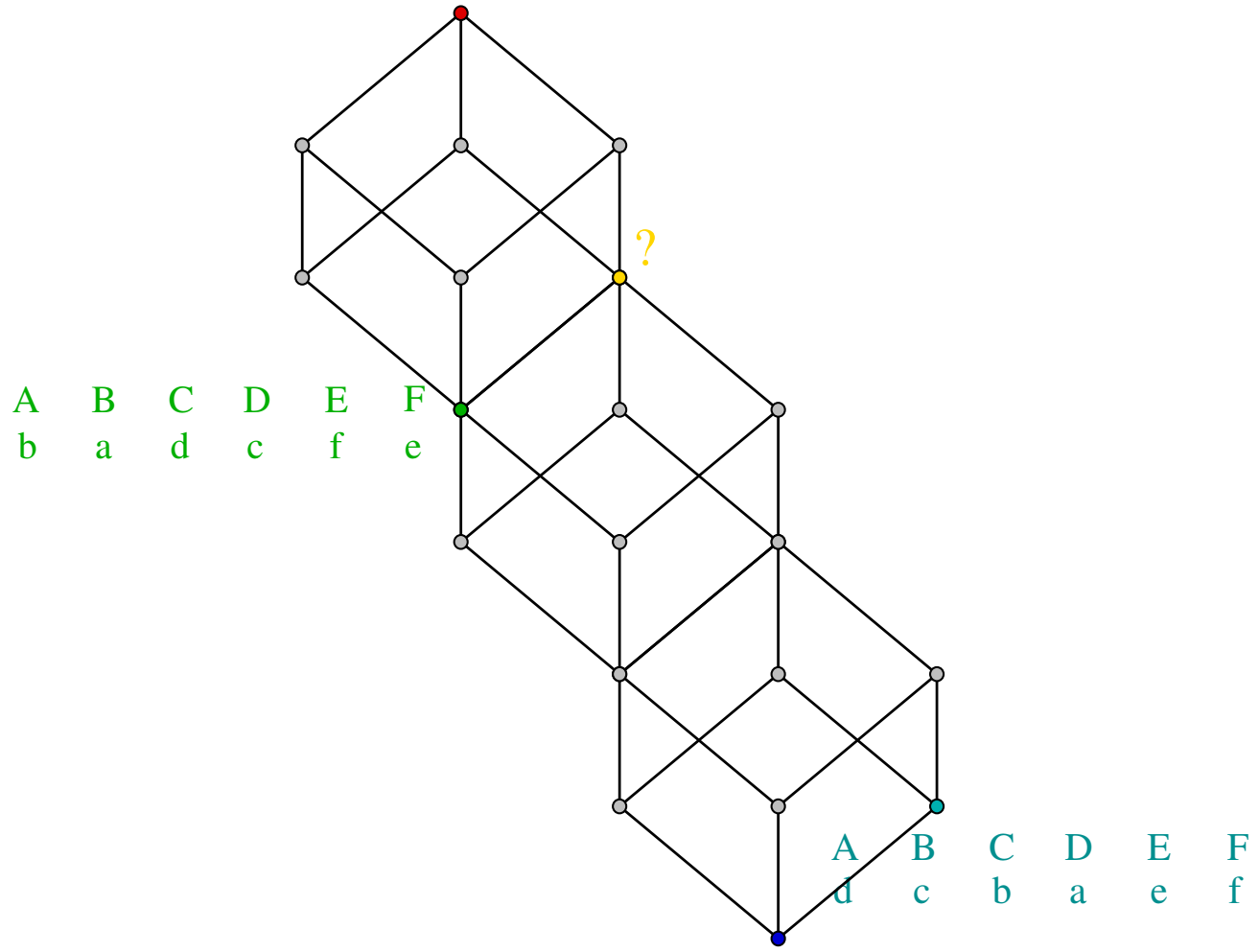

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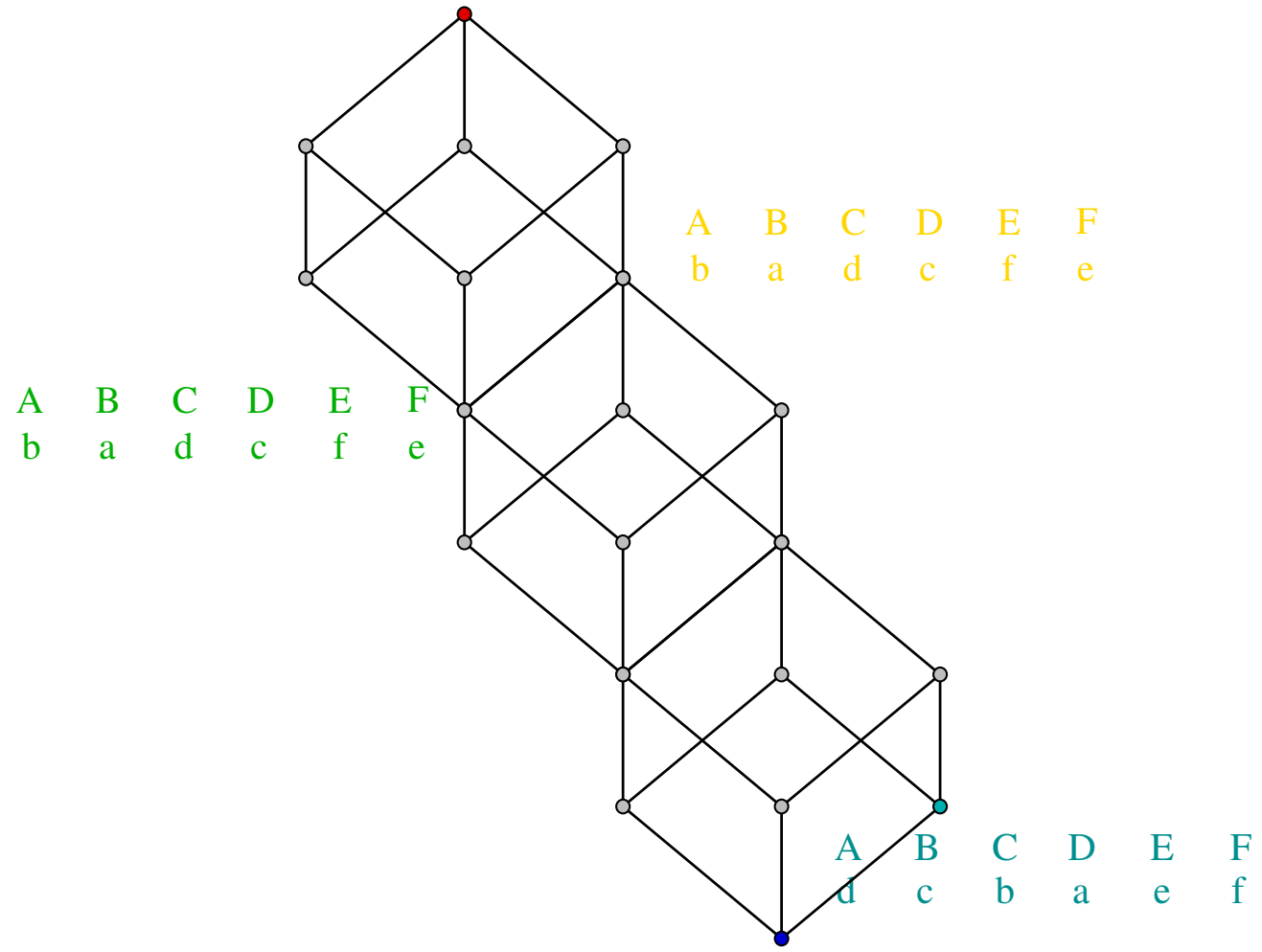
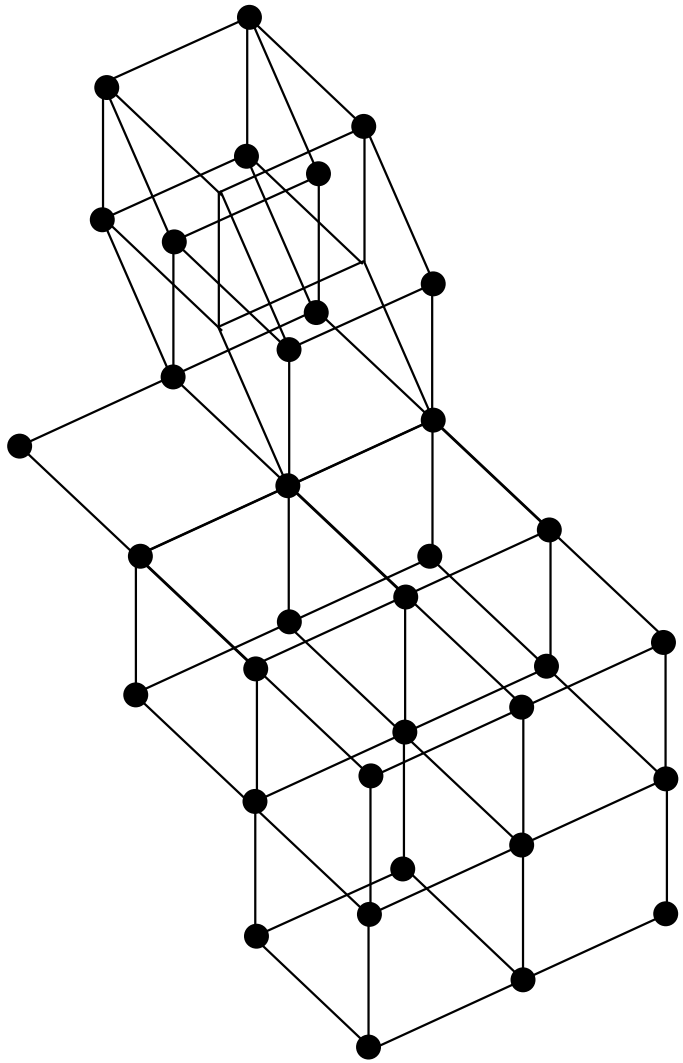


Die Struktur

Satz.
Der Menge der stabilen Hochzeiten kann in einem distributiven Verband organisiert werden.

Jeder distributive Verband entsteht auf diese Weise.



Ende.

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Vielen Dank.