8

Introduction to Greedoids

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8.1. Introduction

Greedoids were invented around 1980 by B. Korte and L. Lovász. Originally, the main motivation for proposing this generalization of the matroid concept came from combinatorial optimization. Korte and Lovász had observed that the optimality of a 'greedy' algorithm could in several instances be traced back to an underlying combinatorial structure that was not a matroid – but (as they named it) a 'greedoid'. In subsequent research greedoids have been shown to be interesting also from various non-algorithmic points of view.

The basic distinction between greedoids and matroids is that greedoids are modeled on the *algorithmic construction* of certain sets, which means that the *ordering of elements* in a set plays an important role. Viewing such ordered sets as words, and the collection of words as a formal language, we arrive at the general definition of a greedoid as a finite language that is closed under the operation of taking initial substrings and satisfies a matroid-type exchange axiom. It is a pleasant feature that greedoids can also be characterized in terms of set systems (the unordered version), but the language formulation (the ordered version) seems more fundamental.

Consider, for instance, the algorithmic construction of a spanning tree in a connected graph. Two simple strategies are: (1) pick one edge at a time, making sure that the current edge does not form a circuit with those already chosen; (2) pick one edge at a time, starting at some given node, so that the current edge connects a visited node with an unvisited node. These well known strategies are used respectively in Kruskal's and in Prim's minimal spanning tree algorithms. In both cases, the collection of feasible sequences of edges, i.e. sequences that are generated by the allowed strategy, forms a greedoid. However, in the first case, but not in the second, any permutation of a feasible sequence of edges is also feasible, so that ordering is irrelevant.