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Connected rigidity matroids and unique realizations of graphs

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Abstract

A d-dimensional *framework* is a straight line realization of a graph G in $\mathbb{R}^d$. We shall only consider *generic* frameworks, in which the co-ordinates of all the vertices of G are algebraically independent. Two frameworks for G are *equivalent* if corresponding edges in the two frameworks have the same length. A framework is a *unique realization* of G in $\mathbb{R}^d$ if every equivalent framework can be obtained from it by an isometry of $\mathbb{R}^d$. Bruce Hendrickson proved that if G has a unique realization in $\mathbb{R}^d$ then G is $(d+1)$-connected and redundantly rigid. He conjectured that every realization of a $(d+1)$-connected and redundantly rigid graph in $\mathbb{R}^d$ is unique. This conjecture is true for $d = 1$ but was disproved by Robert Connelly for $d \geqslant 3$. We resolve the remaining open case by showing that Hendrickson's conjecture is true for $d = 2$. As a corollary we deduce that every realization of a 6-connected graph as a two-dimensional generic framework is a unique realization. Our proof is based on a new inductive characterization of 3-connected graphs whose rigidity matroid is connected.

1. Introduction

We shall consider finite graphs without loops, multiple edges or isolated vertices. A d-dimensional *framework* is a pair (G, p), where $G = (V, E)$ is a graph and p is a map

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