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Matroids and Rigid Structures

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Many engineering problems lead to a system of linear equations – a represented matroid – whose rank controls critical qualitative features of the example (Sugihara, 1984; 1985; White & Whiteley, 1983). We will outline a selection of such matroids, drawn from recent work on the rigidity of spatial structures, reconstruction of polyhedral pictures, and related geometric problems.

For these situations, the combinatorial pattern of the example determines a sparse matrix pattern that has both a generic rank, for general 'independent' values of the non-zero entries, and a geometric rank, for special values for the coordinates of the points, lines, and planes of the corresponding geometric model. Increasingly, the generic rank of these examples has been studied by matroid theoretic techniques. These geometric models provide nice illustrations and applications of techniques such as matroid union, truncation, and semimodular functions. The basic unsolved problems in these examples highlight certain unsolved problems in matroid theory. Their study should also lead to new results in matroid theory.

1.1. Bar Frameworks on the Line – the Graphic Matroid

We begin with the simplest example, which will introduce the vocabulary and the basic pattern. We place a series of distinct points on a line, and specify certain *bars* – pairs of joints which are to maintain their distance – defining a *bar framework on the line*. We ask whether the entire framework is 'rigid' – i.e. does any motion of the joints along the line, preserving these distances, give all joints the same velocity, acceleration, etc.? Clearly a framework has an *underlying graph* $G = (V, E)$, with a vertex v_i for each joint p_i and an undirected edge $\{i, j\}$ for each bar $\{p_i, p_j\}$. In fact, we describe the framework as $G(\mathbf{p})$, where G is a graph without multiple edges or loops, and $\mathbf{p}$ is an assignment of points p_i to the vertices v_i. If this graph is not connected,