6 ORIENTED MATROIDS

Jürgen Richter-Gebert and Günter M. Ziegler

INTRODUCTION

The theory of *oriented matroids* provides a broad setting in which to model, describe, and analyze combinatorial properties of geometric configurations. Mathematical objects of study that appear to be disjoint and independent, such as *point and vector configurations*, *arrangements of hyperplanes*, *convex polytopes*, *directed graphs*, and *linear programs* find a common generalization in the language of oriented matroids.

The oriented matroid of a finite set of points P extracts relative position and orientation information from the configuration; for example, it can be given by a list of signs that encodes the orientations of all the bases of P. In the passage from a concrete point configuration to its oriented matroid, metrical information is lost, but many structural properties of P have their counterparts at the—purely combinatorial—level of the oriented matroid. (In computational geometry, the oriented matroid data of an unlabelled point configuration are sometimes called the *order type*.) From the oriented matroid of a configuration of points, one can compute not only that face lattice of the convex hull, but also the set of all its triangulations and subdivisions (cf. Chapter 16).

We first introduce oriented matroids in the context of several models and motivations (Section 6.1). Then we present some equivalent axiomatizations (Section 6.2). Finally, we discuss concepts that play central roles in the theory of oriented matroids (Section 6.3), among them *duality*, *realizability*, the study of *simplicial cells*, and the treatment of *convexity*.

6.1 MODELS AND MOTIVATIONS

This section discusses geometric examples that are usually treated on the level of concrete coordinates, but where an "oriented matroid point of view" gives deeper insight. We also present these examples as standard models that provide intuition for the behavior of general oriented matroids.

6.1.1 ORIENTED BASES OF VECTOR CONFIGURATIONS

GLOSSARY

Vector configuration X: A matrix $X = (x_1, \ldots, x_n) \in (\mathbb{R}^d)^n$, usually assumed to have full rank d.

Preliminary version (August 10, 2017). To appear in the Handbook of Discrete and Computational Geometry,
J.E. Goodman, J. O'Rourke, and C. D. Tóth (editors), 3rd edition, CRC Press, Boca Raton, FL, 2017.