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ON THEOREMS OF WHITNEY AND TUTTE

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Short proofs of two theorems are given: (i) Whitney's 2-isomorphism theorem characterizing all graphs with the same cycle matroid, and (ii) Tutte's excluded minor characterization of those binary matroids that are graphic. Graph connectivity plays an important role in both proofs.

1. Introduction

Familiarity with graph and matroid theory is assumed; see [1] and [10]. Where G is a graph with $S \subseteq E(G)$, $G[S]$ denotes the subgraph induced by S. A partition $\{S, T\}$ of $E(G)$ is a *k-separation* of G, for k a positive integer, if $|S| \geq k \leq |T|$ and $|V(G[S]) \cap V(G[T])| \leq k$. A graph is *n-connected*, for n a positive integer, if it has no k-separation for $k < n$; a 2-connected graph is *nonseparable*.

Let G be a nonseparable graph with 2-separation $\{S, T\}$ and let $V(G[S]) \cap V(G[T]) = \{x, y\}$. Let G' be the graph obtained from G by interchanging in $G[S]$ the incidences of the edges at x and y. Then G' is obtained from G by *reversing* $G[S]$. A graph obtainable from G by a sequence of reversals is *2-isomorphic* to G.

Let $M(G)$ denote the cycle matroid of G. Whitney [12] proved the following result.

(1.1) Let G and G' be nonseparable graphs. Then $M(G) = M(G')$ if and only if G and G' are 2-isomorphic.

Let K_5 and $K_{3,3}$ denote the Kuratowski graphs and let F_7 denote the Fano matroid. Denote the dual of a matroid M by M^*.

Tutte [8] proved the following result.

(1.2) Let M be a binary matroid. Then M is graphic if and only if M has no F_7, F_7^*, $M^*(K_5)$ or $M^*(K_{3,3})$ minor.

The purpose of this paper is to provide new short proofs of (1.1) and (1.2). It is hoped the present proofs will be more accessible and will provide additional insight into the results. Graph connectivity plays an important role in both proofs.

The paper is outlined as follows: (1.1) is proved in Section 2, some preliminary

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