A Short Proof of Tutte's
Characterization of Totally Unimodular Matrices

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Dedicated to Alan J. Hoffman on the occasion of his 65th birthday.

ABSTRACT

We give, in terms of totally unimodular matrices, a short and easy proof of Tutte's characterization of regular matroids.

1. INTRODUCTION

We give a short and easy proof of the following well-known result of Tutte (1958, 1965, 1971):

TUTTE'S THEOREM. *Let A be a $\{0,1\}$-matrix. Then the following are equivalent*:

(i) *A has a totally unimodular signing,*

(ii) *A cannot be transformed to*

$$M(F_7) := \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix}$$

by applying (repeatedly) the following operations:

(1)
> *deleting rows or columns,*
> *permuting rows or columns,*
> *taking the transposed matrix,*
> *pivoting over* GF(2).